# PyDynamic Documentation

*Release 1.2.1*

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# Contents


1 Getting started 3

2 Evaluation of uncertainties 27

3 Design of deconvolution filters 39

4 Miscellaneous 43

5 Indices and tables 53

6 References 55

Bibliography 57

Python Module Index 59

Index 61

PyDynamic is a Python software package developed jointly by mathematicians from Physikalisch-Technische Bundesanstalt (Germany) and National Physical Laboratory (UK) as part of the joint European Research Project EMPIR 14SIP08 Dynamic.

For the PyDynamic homepage go to GitHub.

*PyDynamic* is written in Python 3.

Contents:

CHAPTER 1

# Getting started

## 1.1 Installation

**If you just want to use the software, the easiest way is to run from your system's command line** pip install PyDynamic

This will download the latest version from the Python package repository and copy it into your local folder of third-party libraries. Usage in any Python environment on your computer is then possible by

```
import PyDynamic
```

or, for example, for the module containing the Fourier domain uncertainty methods:

```
from PyDynamic.uncertainty import propagate_DFT
```

**Updates of the software can be installed via** pip install –upgrade PyDynamic

For collaboration we recommend using Github Desktop or any other git-compatible version control software and cloning the repository. In this way, any updates to the software will be highlighted in the version control software and can be applied very easily.

If you have downloaded this software, we would be very thankful for letting us know. You may, for instance, drop an email to one of the authors (e.g. Sascha Eichstädt or Ian Smith)

## 1.2 Quick Examples

On the project website you can find various examples illustrating the application of the software in the examples folder. Here is just a short list to get you started.

Uncertainty propagation for the application of a FIR filter with coefficients $b$ with which an uncertainty $ub$ is associated. The filter input signal is $x$ with known noise standard deviation $sigma$. The filter output signal is $y$ with associated uncertainty $uy$.

```
from PyDynamic.uncertainty.propagate_filter import FIRuncFilter
y, uy = FIRuncFilter(x, sigma, b, ub)
```

Uncertainty propagation through the application of the discrete Fourier transform (DFT). The time domain signal is *x* with associated squared uncertainty *ux*. The result of the DFT is the vector *X* of real and imaginary parts of the DFT applied to *x* and the associated uncertainty *UX*.

```
from PyDynamic.uncertainty.propagate_DFT import GUM_DFT
X, UX = GUM_DFT(x, ux)
```

Sequential application of the Monte Carlo method for uncertainty propagation for the case of filtering a time domain signal *x* with an IIR filter *b,a* with uncertainty associated with the filter coefficients *Uab* and signal noise standard deviation *sigma*. The filter output is the signal *y and the Monte Carlo method calculates point-wise uncertainties *uy* and coverage intervals *Py* corresponding to the specified percentiles.

```
from PyDynamic.uncertainty.propagate_MonteCarlo import SMC
y, uy, Py = SMC(x, sigma, b, a, Uab, runs=1000, Perc=[0.025,0.975])
```

## 1.3 Detailed examples

```
%pylab inline
import numpy as np
import scipy.signal as dsp
from palettable.colorbrewer.qualitative import Dark2_8

colors = Dark2_8.mpl_colors
rst = np.random.RandomState(1)
```

```
Populating the interactive namespace from numpy and matplotlib
```

### 1.3.1 Design of a digital deconvolution filter (FIR type)

```
from PyDynamic.deconvolution.fit_filter import LSFIR_unc
from PyDynamic.misc.SecondOrderSystem import *
from PyDynamic.misc.testsignals import shocklikeGaussian
from PyDynamic.misc.filterstuff import kaiser_lowpass, db
from PyDynamic.uncertainty.propagate_filter import FIRuncFilter
from PyDynamic.misc.tools import make_semiposdef
```

```
# parameters of simulated measurement
Fs = 500e3
Ts = 1 / Fs

# sensor/measurement system
f0 = 36e3; uf0 = 0.01*f0
S0 = 0.4; uS0= 0.001*S0
delta = 0.01; udelta = 0.1*delta

# transform continuous system to digital filter
bc, ac = sos_phys2filter(S0,delta,f0)
b, a = dsp.bilinear(bc, ac, Fs)
```

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```
# Monte Carlo for calculation of unc. assoc. with [real(H),imag(H)]
f = np.linspace(0, 120e3, 200)
Hfc = sos_FreqResp(S0, delta, f0, f)
Hf = dsp.freqz(b,a,2*np.pi*f/Fs)[1]

runs = 10000
MCS0 = S0 + rst.randn(runs)*uS0
MCd  = delta+ rst.randn(runs)*udelta
MCf0 = f0 + rst.randn(runs)*uf0
HMC = np.zeros((runs, len(f)),dtype=complex)
for k in range(runs):
    bc_,ac_ = sos_phys2filter(MCS0[k], MCd[k], MCf0[k])
    b_,a_ = dsp.bilinear(bc_,ac_,Fs)
    HMC[k,:] = dsp.freqz(b_,a_,2*np.pi*f/Fs)[1]

H = np.r_[np.real(Hf), np.imag(Hf)]
uAbs = np.std(np.abs(HMC),axis=0)
uPhas= np.std(np.angle(HMC),axis=0)
UH= np.cov(np.hstack((np.real(HMC),np.imag(HMC))),rowvar=0)
UH= make_semiposdef(UH)
```

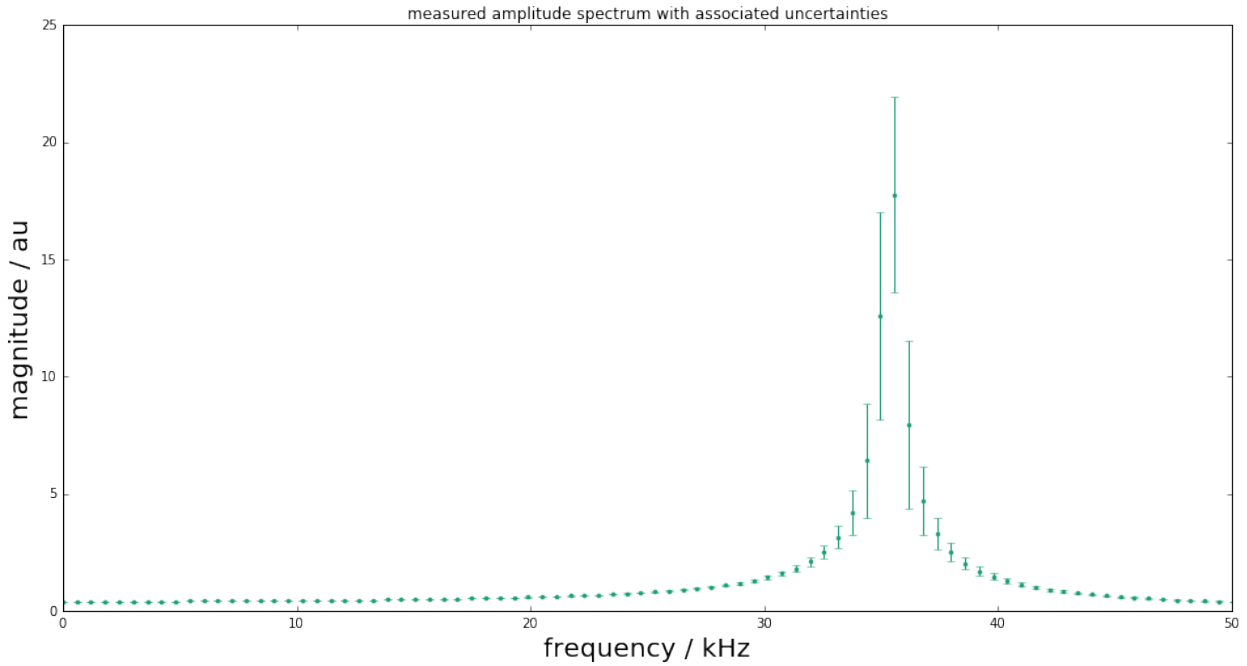
### Problem description

Assume information about a linear time-invariant (LTI) measurement system to be available in terms of its frequency response values $H(j\omega)$ at a set of frequencies together with associated uncertainties:

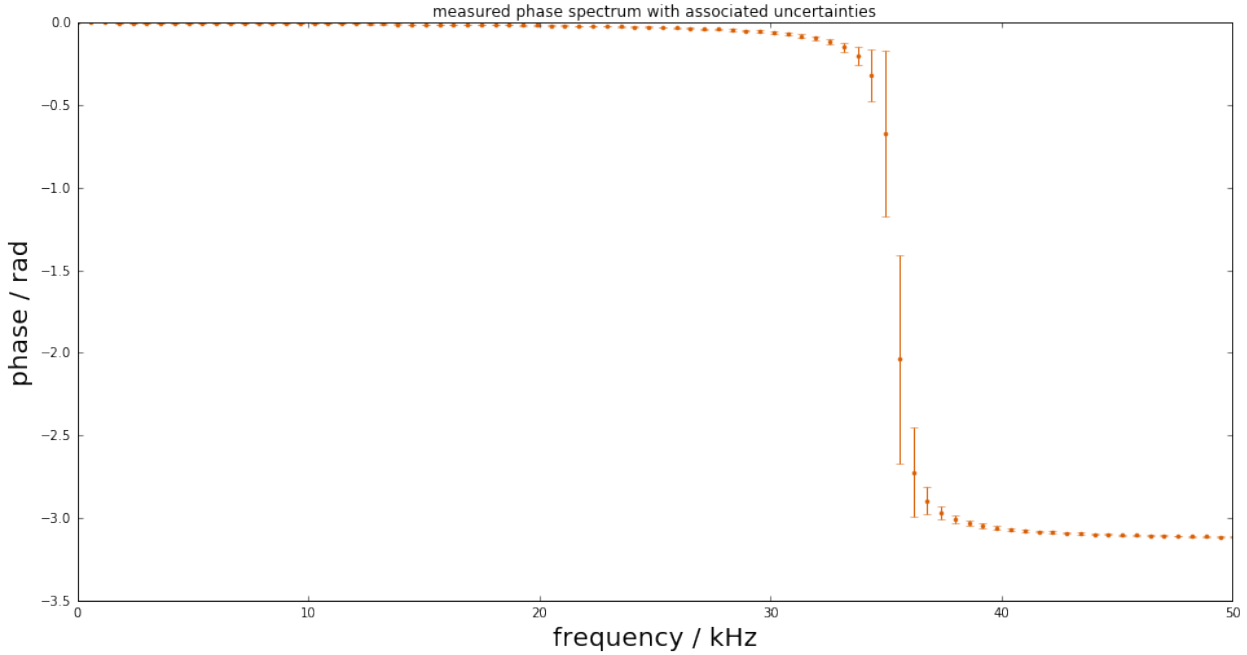
$$\mathbf{H} = (|H(j\omega_1)|, \ldots, |H(j\omega_N)|, \angle H(j\omega_1), \ldots, \angle H(j\omega_N)) \tag{1.1}$$

$$u(\mathbf{H}) = (u(|H(j\omega_1)|), \ldots, u(|H(j\omega_N)|), u(\angle H(j\omega_1)), \ldots, u(\angle H(j\omega_N)))$$

```
figure(figsize=(16,8))
errorbar(f*1e-3, np.abs(Hf), uAbs, fmt=".", color=colors[0])
title("measured amplitude spectrum with associated uncertainties")
xlim(0,50)
xlabel("frequency / kHz",fontsize=20)
ylabel("magnitude / au",fontsize=20);
```




```
figure(figsize=(16,8))
errorbar(f*1e-3, np.angle(Hf), uPhas, fmt=".", color=colors[1])
title("measured phase spectrum with associated uncertainties")
xlim(0,50)
xlabel("frequency / kHz",fontsize=20)
ylabel("phase / rad",fontsize=20);
```



### Simulated measurement

Measurements with this system are then modeled as a convolution of the system's impulse response

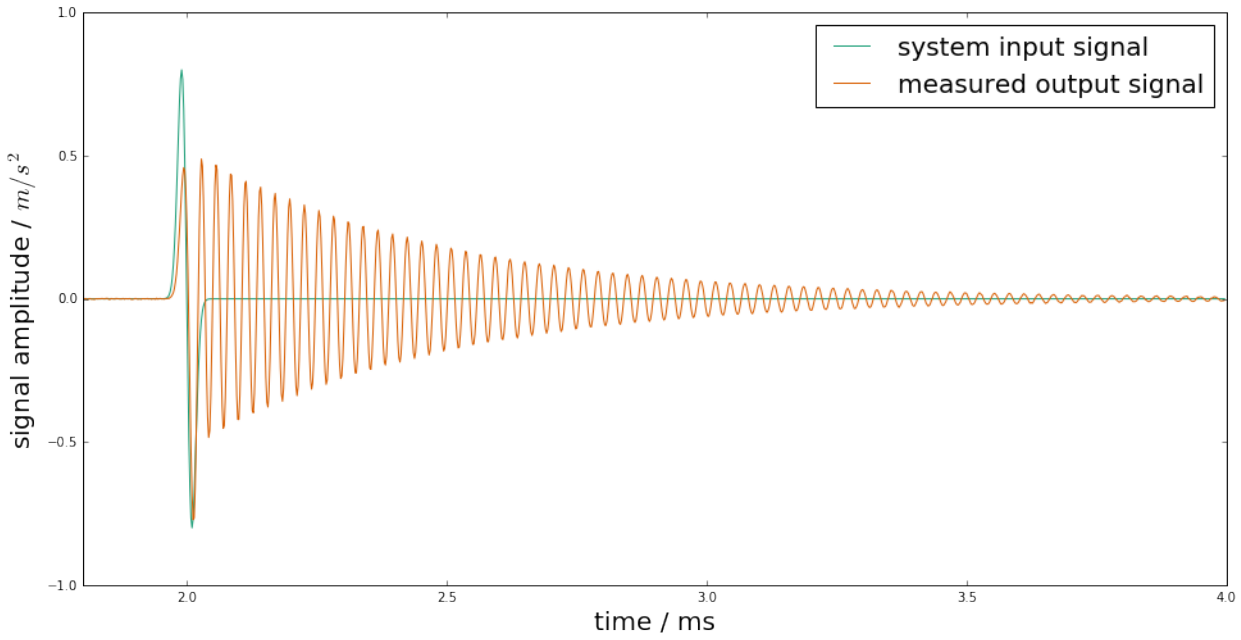
$$h(t) = \mathcal{F}^{-1}(H(j\omega))$$

with the input signal $x(t)$, after an analogue-to-digital conversion producing the measured signal

$$y[n] = (h * x)(t_n) \qquad n = 1, \ldots, M$$

```
# simulate input and output signals
time = np.arange(0, 4e-3 - Ts, Ts)
#x = shocklikeGaussian(time, t0 = 2e-3, sigma = 1e-5, m0=0.8)
m0 = 0.8; sigma = 1e-5; t0 = 2e-3
x = -m0*(time-t0)/sigma * np.exp(0.5)*np.exp(-(time-t0) ** 2 / (2 * sigma ** 2))
y = dsp.lfilter(b, a, x)
noise = 1e-3
yn = y + rst.randn(np.size(y)) * noise
```

```
figure(figsize=(16,8))
plot(time*1e3, x, label="system input signal", color=colors[0])
plot(time*1e3, yn,label="measured output signal", color=colors[1])
legend(fontsize=20)
xlim(1.8,4); ylim(-1,1)
xlabel("time / ms",fontsize=20)
ylabel(r"signal amplitude / $m/s^2$",fontsize=20);
```




### Design of the deconvolution filter

The aim is to derive a digital filter with finite impulse response (FIR)

$$g(z) = \sum_{k=0}^{K} b_k z^{-k}$$

such that the filtered signal

$$\hat{x}[n] = (g * y)[n] \qquad n = 1, \ldots, M$$

<<<<<<< HEAD is an estimate of the system's input signal at the discrete time points ======= is an estimate of the system's input signal at the discrete time points. >>>>>>> devel1

Publication

- Elster and Link "Uncertainty evaluation for dynamic measurements modelled by a linear time-invariant system" Metrologia, 2008
- Vuerinckx R, Rolain Y, Schoukens J and Pintelon R "Design of stable IIR filters in the complex domain by automatic delay selection" IEEE Trans. Signal Process. 44 2339–44, 1996

Determine FIR filter coefficients such that

$$H(j\omega)g(e^{j\omega/F_s}) \approx e^{-j\omega n_0/F_s} \qquad \text{for} \qquad |\omega| \leq \omega_1$$

with a pre-defined time delay $n_0$ to improve the fit quality (typically half the filter order).

Consider as least-squares problem

$$(y - Xb)^T W^{-1} (y - Xb)$$

with - $y$ real and imaginary parts of the *reciprocal* and phase shifted measured frequency response values - $X$ the model matrix with entries $e^{-jk\omega/Fs}$ - $b$ the sought FIR filter coefficients - $W$ a weighting matrix (usually derived from the uncertainties associated with the frequency response measurements

Filter coefficients and associated uncertainties are thus obtained as

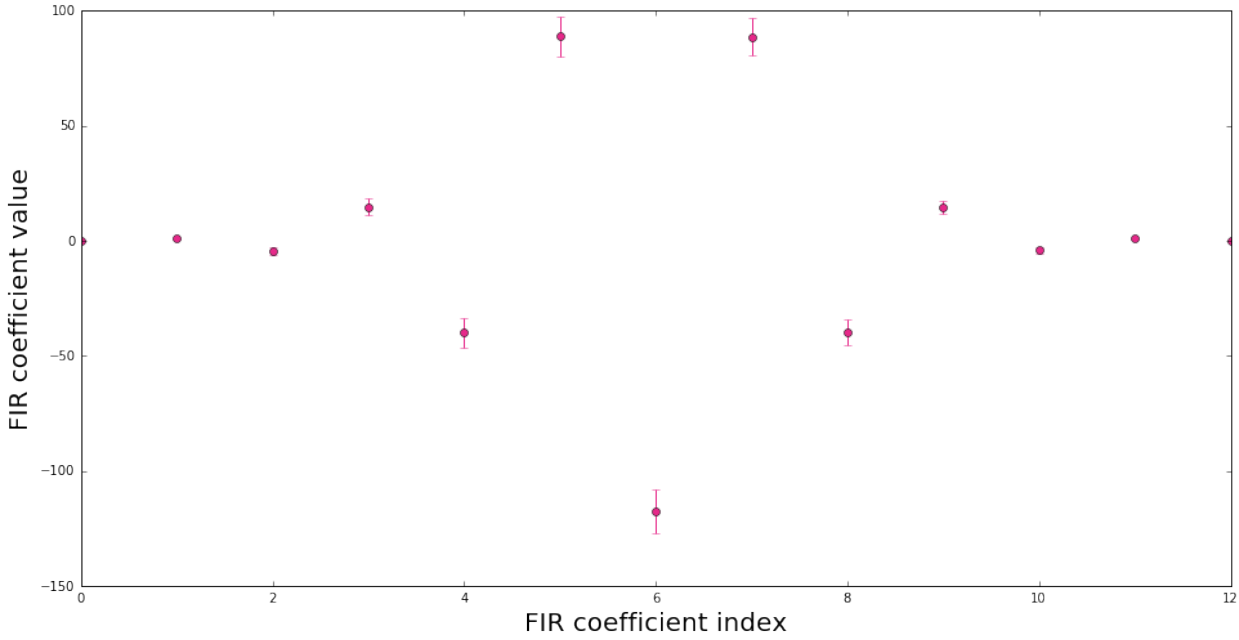
$$b = \left(X^T W^{-1} X\right)^{-1} X^T W^{-1} y$$

$$u_b = \left(X^T W^{-1} X\right)^{-1} X^T W^{-1} U_y W^{-1} X \left(X^T W^{-1} X\right)^{-1}$$

```
# Calculation of FIR deconvolution filter and its assoc. unc.
N = 12; tau = N//2
bF, UbF = deconv.LSFIR_unc(H,UH,N,tau,f,Fs)
```

```
Least-squares fit of an order 12 digital FIR filter to the
reciprocal of a frequency response given by 400 values
and propagation of associated uncertainties.
Final rms error = 1.545423e+01
```

```
figure(figsize=(16,8))
errorbar(range(N+1), bF, np.sqrt(np.diag(UbF)), fmt="o", color=colors[3])
xlabel("FIR coefficient index", fontsize=20)
ylabel("FIR coefficient value", fontsize=20);
```




In order to render the ill-posed estimation problem stable, the FIR inverse filter is accompanied with an FIR low-pass filter.

Application of the deconvolution filter for input estimation is then carried out as

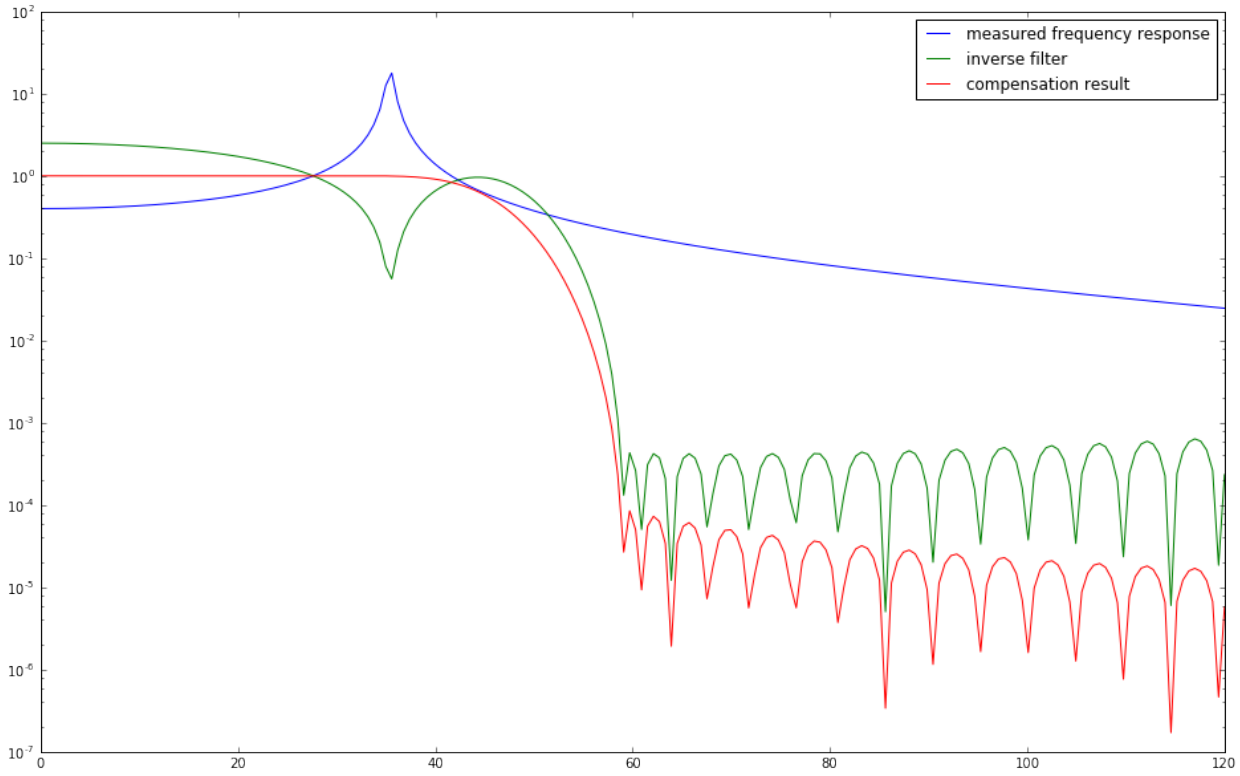
$$\hat{x}[n - n_0] = (g * (g_{low} * y)[n]$$

with point-wise associated uncertainties calculated as

$$u^2(\hat{x}[n - n_0] = b^T U_{x_{low}[n]} b + x_{low}^T[n] U_b x_{low}[n] + trace(U_{x_{low}[n]} U_b)$$

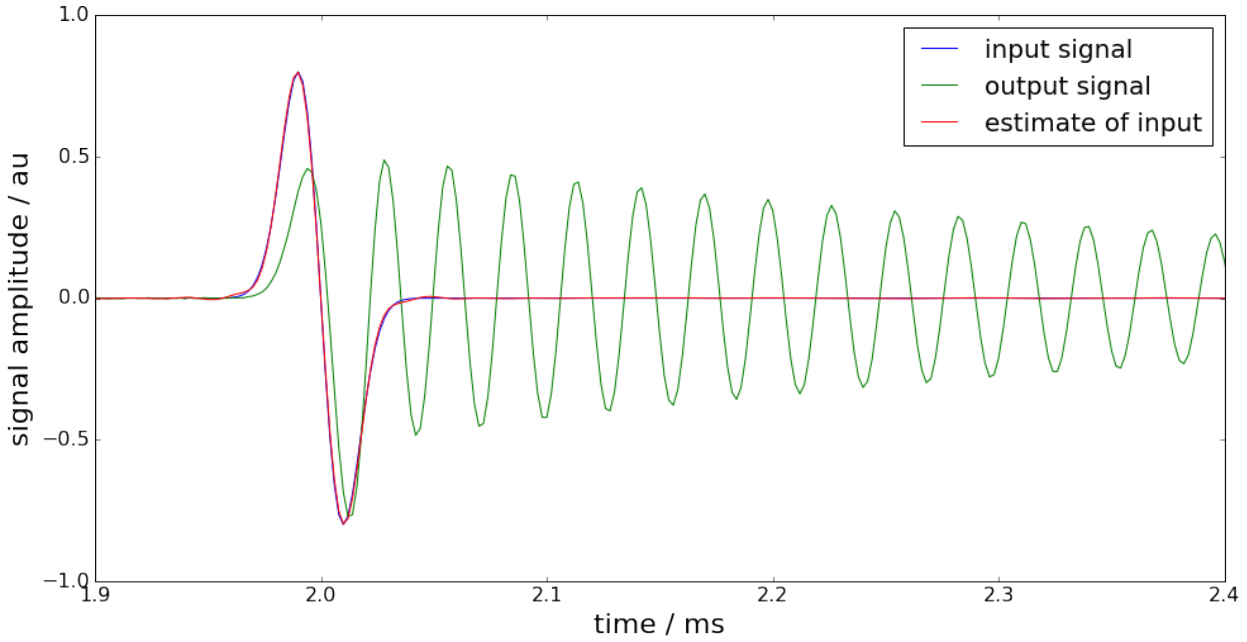
```
fcut = f0+10e3; low_order = 100
blow, lshift = kaiser_lowpass(low_order, fcut, Fs)
shift = -tau - lshift
```

```
figure(figsize=(16,10))
HbF = dsp.freqz(bF,1,2*np.pi*f/Fs)[1]*dsp.freqz(blow,1,2*np.pi*f/Fs)[1]
semilogy(f*1e-3, np.abs(Hf), label="measured frequency response")
semilogy(f*1e-3, np.abs(HbF),label="inverse filter")
semilogy(f*1e-3, np.abs(Hf*HbF), label="compensation result")
legend();
```

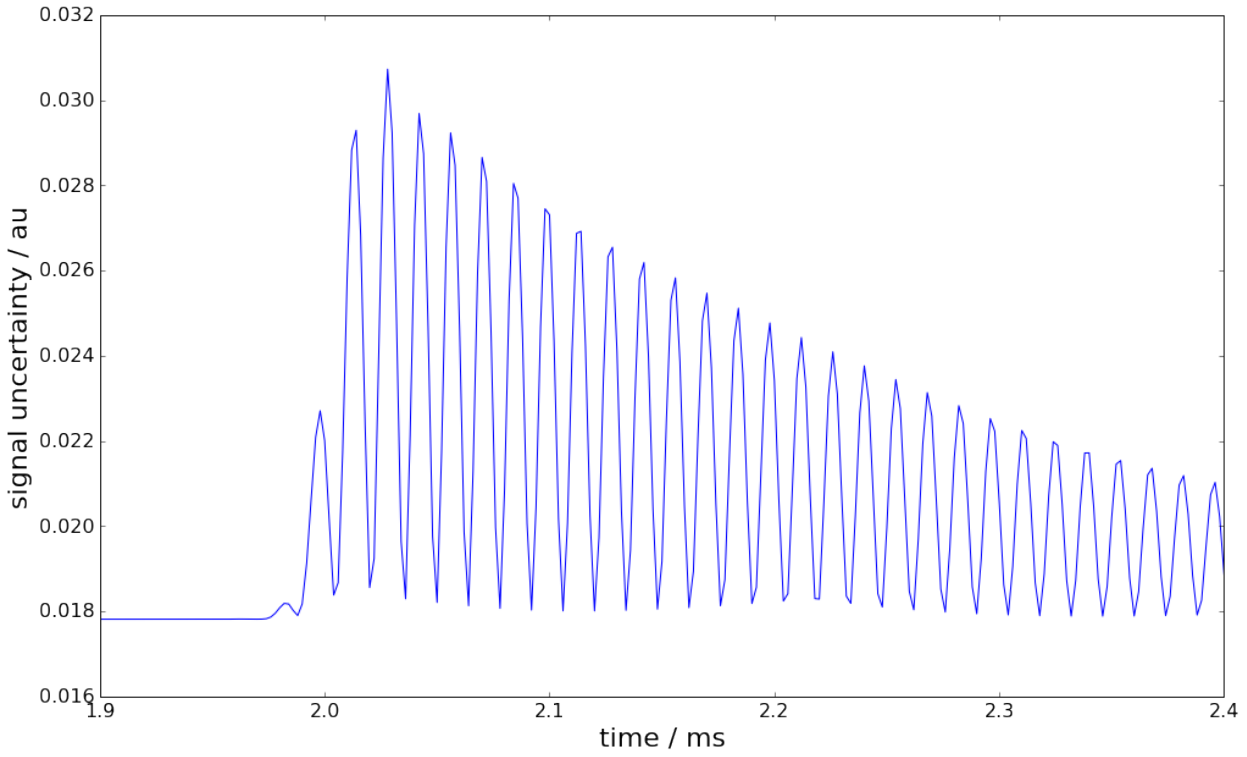



```
xhat,Uxhat = FIRuncFilter(yn,noise,bF,UbF,shift,blow)
```

```
figure(figsize=(16,8))
plot(time*1e3,x, label='input signal')
plot(time*1e3,yn,label='output signal')
plot(time*1e3,xhat,label='estimate of input')
legend(fontsize=20)
xlabel('time / ms',fontsize=22)
ylabel('signal amplitude / au',fontsize=22)
tick_params(which="both",labelsize=16)
xlim(1.9,2.4); ylim(-1,1);
```




```
figure(figsize=(16,10))
plot(time*1e3,Uxhat)
xlabel('time / ms',fontsize=22)
ylabel('signal uncertainty / au',fontsize=22)
subplots_adjust(left=0.15,right=0.95)
tick_params(which='both', labelsize=16)
xlim(1.9,2.4);
```



### Basic workflow in PyDynamic

Fit an FIR filter to the reciprocal of the measured frequency response

```
from PyDynamic.deconvolution.fit_filter import LSFIR_unc
bF, UbF = LSFIR_unc(H,UH,N,tau,f,Fs, verbose=False)
```

with - `H` the measured frequency response values - `UH` the covariance (i.e. uncertainty) associated with real and imaginary parts of H - `N` the filter order - `tau` the filter delay in samples - `f` the vector of frequencies at which H is given - `Fs` the sampling frequency for the digital FIR filter

Propagate the uncertainty associated with the measurement noise and the FIR filter through the deconvolution process

```
xhat,Uxhat = FIRuncFilter(yn,noise,bF,UbF,shift,blow)
```

with - `yn` the noisy measurement - `noise` the std of the noise - `shift` the total delay of the FIR filter and the low-pass filter - `blow` the coefficients of the FIR low-pass filter

```
%pylab inline
import scipy.signal as dsp
```

```
Populating the interactive namespace from numpy and matplotlib
```

## 1.3.2 Uncertainty propagation for IIR filters

```
from PyDynamic.misc.testsignals import rect
from PyDynamic.uncertainty.propagate_filter import IIRuncFilter
from PyDynamic.uncertainty.propagate_MonteCarlo import SMC
from PyDynamic.misc.tools import make_semiposdef
```

Digital filters with infinite impulse response (IIR) are a common tool in signal processing. Consider the measurand to be the output signal of an IIR filter with z-domain transfer function

$$G(z) = \frac{\sum_{n=0}^{N_b} b_n z^{-n}}{1 + \sum_{m=1}^{N_a} a_m z^{-m}}.$$

The measurement model is thus given by

$$y[k] = \sum_{n=0}^{N_b} b_n x[k-n] - \sum_{m=1}^{N_a} a_m y[k-m]$$

As input quantities to the model the input signal values $x[k]$ and the IIR filter coefficients $(b_0, \ldots, a_{N_a})$ are considered.

### Linearisation-based uncertainty propagation

Scientific publication

```
A. Link and C. Elster,
"Uncertainty evaluation for IIR filtering using a
state-space approach,"
Meas. Sci. Technol., vol. 20, no. 5, 2009.
```

The linearisation method for the propagation of uncertainties through the IIR model is based on a state-space model representation of the IIR filter equation

$$z^\top[n+1] = \begin{pmatrix} -a_1 & \cdots & \cdots & -a_{N_a} \\ 0 & & & \\ \vdots & & I_{N_a-1} & \\ 0 & & & \end{pmatrix} z^\top[n] + \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} X[n], \tag{1.2}$$

$$y[n] = (d_1, \ldots, d_{N_a}) z^\top[n] + b_0 X[n] + \Delta[n], \tag{1.3}$$

$$y[n] = d^T z[n] + b_0 y[n] \tag{1.4}$$

$$u^2(y[n]) = \phi^T(n) U_\mu \phi(n) + d^T P_z[n] d + b_0^2, \tag{1.5}$$

where

$$\boldsymbol{\phi}(n) = \left( \frac{\partial x[n]}{\partial \mu_1}, \ldots, \frac{\partial x[n]}{\partial \mu_{N+N_a+N_b+1}} \right)^T$$

$$\mathbf{P_z}[n] = \sum_{m<n} \left( \frac{\partial \mathbf{z}[n]}{\partial y[m]} \right) \left( \frac{\partial \mathbf{z}[n]}{\partial y[m]} \right)^T u^2(y[m]).$$

The linearization-based uncertainty propagation method for IIR filters provides

- propagation schemes for white noise and colored noise in the filter input signal
- incorporation of uncertainties in the IIR filter coefficients
- online evaluation of the point-wise uncertainties associated with the IIR filter output

### Implementation in PyDynamic

`y,Uy = IIRuncFilter(x,noise,b,a,Uab)`

with - x the filter input signal sequency - noise the standard deviation of the measurement noise in x - b,a the IIR filter coefficient - Uab, the covariance matrix associated with $(a_1, \ldots, b_{N_b})$

Remark

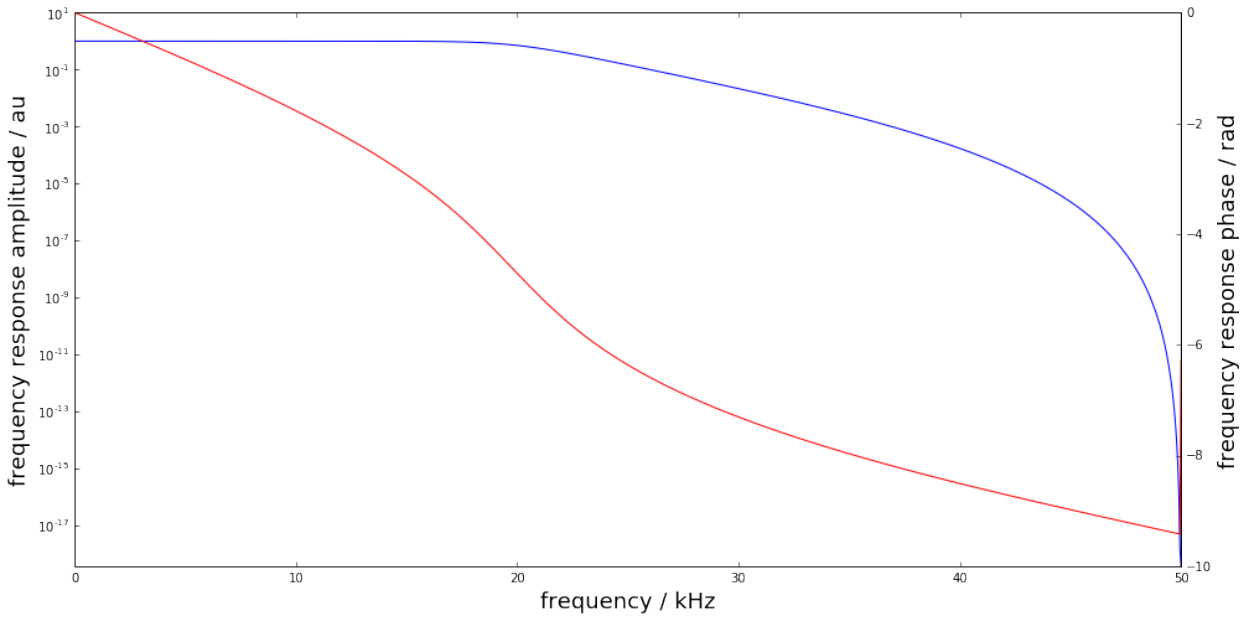
```
Implementation for more general noise processes than white noise is considered for␣
↪one of the next revisions.
```

### Example

```
# parameters of simulated measurement
Fs = 100e3
Ts = 1.0/Fs

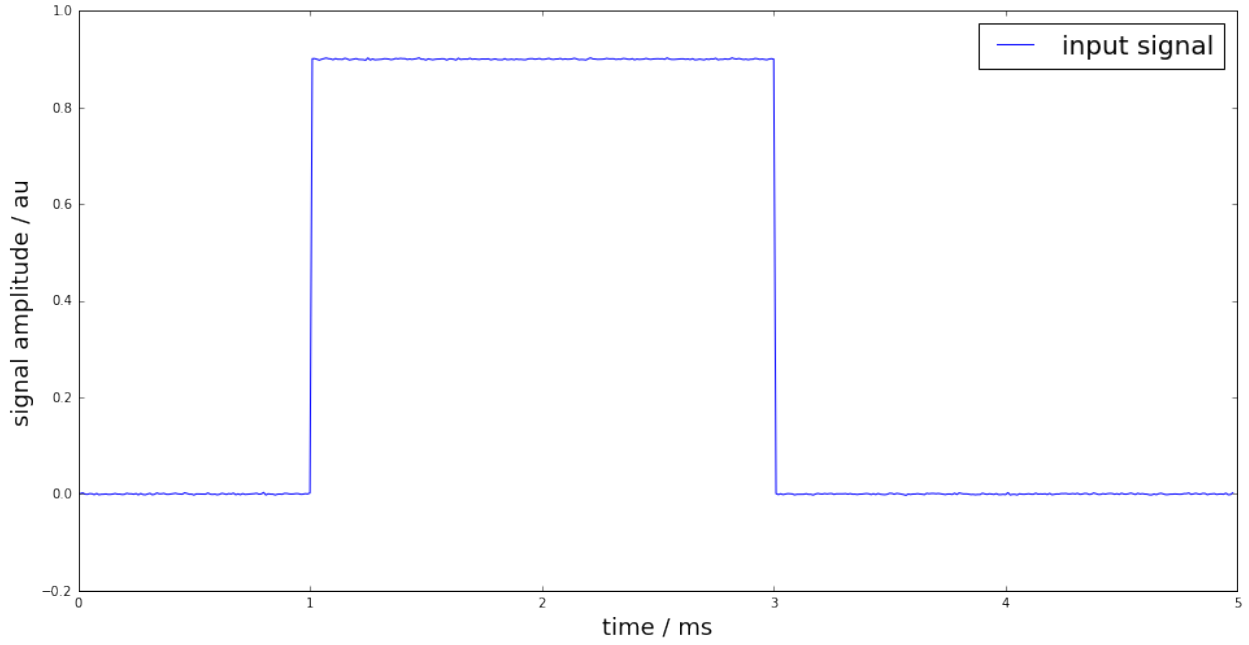
# nominal system parameter
fcut = 20e3
L = 6
b,a = dsp.butter(L,2*fcut/Fs,btype='lowpass')
```

```
f = linspace(0,Fs/2,1000)
figure(figsize=(16,8))
semilogy(f*1e-3, abs(dsp.freqz(b,a,2*np.pi*f/Fs)[1]))
ylim(0,10);
xlabel("frequency / kHz",fontsize=18); ylabel("frequency response amplitude / au",
↪fontsize=18)
ax2 = gca().twinx()
ax2.plot(f*1e-3, unwrap(angle(dsp.freqz(b,a,2*np.pi*f/Fs)[1])),color="r")
ax2.set_ylabel("frequency response phase / rad",fontsize=18);
```




```
time = np.arange(0,499*Ts,Ts)
t0 = 100*Ts; t1 = 300*Ts
height = 0.9
noise = 1e-3
x = rect(time,t0,t1,height,noise=noise)
```

```
figure(figsize=(16,8))
plot(time*1e3, x, label="input signal")
legend(fontsize=20)
xlabel('time / ms',fontsize=18)
ylabel('signal amplitude / au',fontsize=18);
```



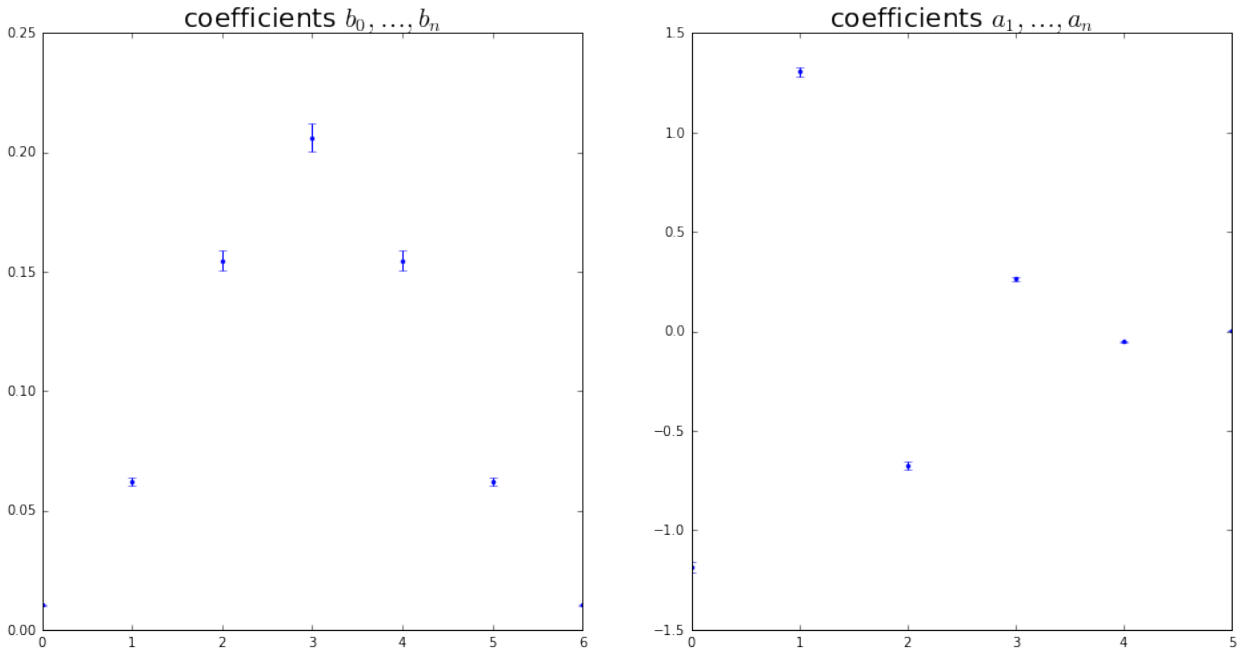

```
# uncertain knowledge: fcut between 19.8kHz and 20.2kHz
runs = 10000
FC = fcut + (2*np.random.rand(runs)-1)*0.2e3
AB = np.zeros((runs,len(b)+len(a)-1))

for k in range(runs):
    bb,aa = dsp.butter(L,2*FC[k]/Fs,btype='lowpass')
    AB[k,:] = np.hstack((aa[1:],bb))

Uab = make_semiposdef(np.cov(AB,rowvar=0))
```

Uncertain knowledge: low-pass cut-off frequency is between 19.8 and 20.2 kHz

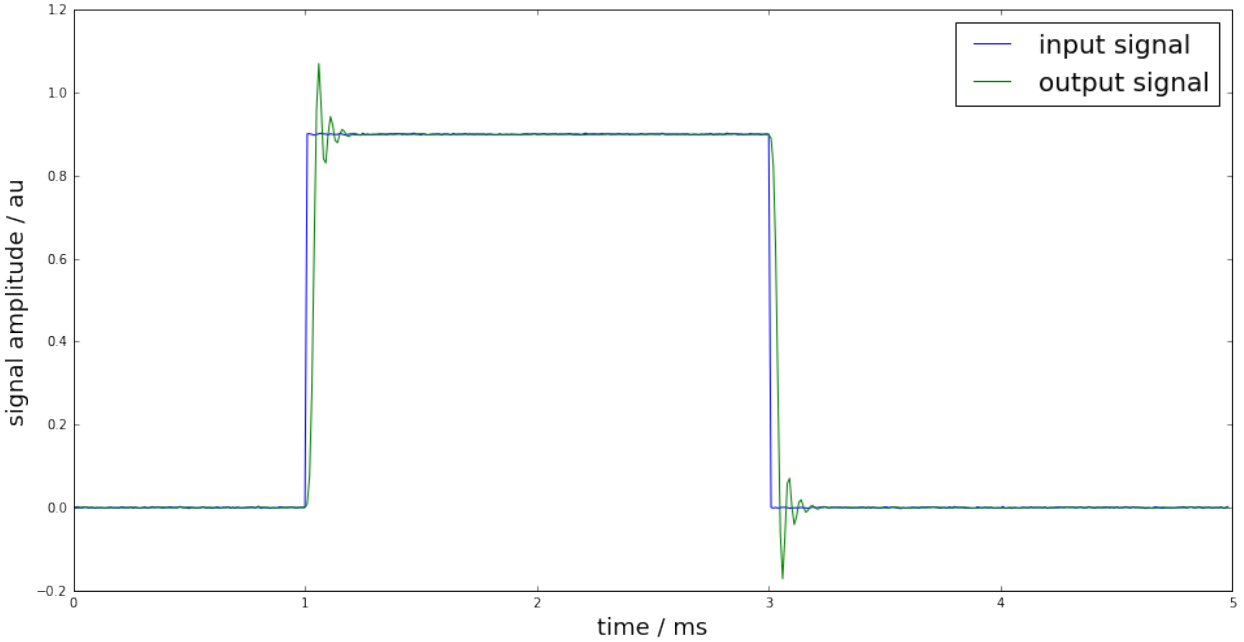
```
figure(figsize=(16,8))
subplot(121)
errorbar(range(len(b)), b, sqrt(diag(Uab[L:,L:])),fmt=".")
title(r"coefficients $b_0,\ldots,b_n$",fontsize=20)
subplot(122)
errorbar(range(len(a)-1), a[1:], sqrt(diag(Uab[:L, :L])),fmt=".");
title(r"coefficients $a_1,\ldots,a_n$",fontsize=20);
```



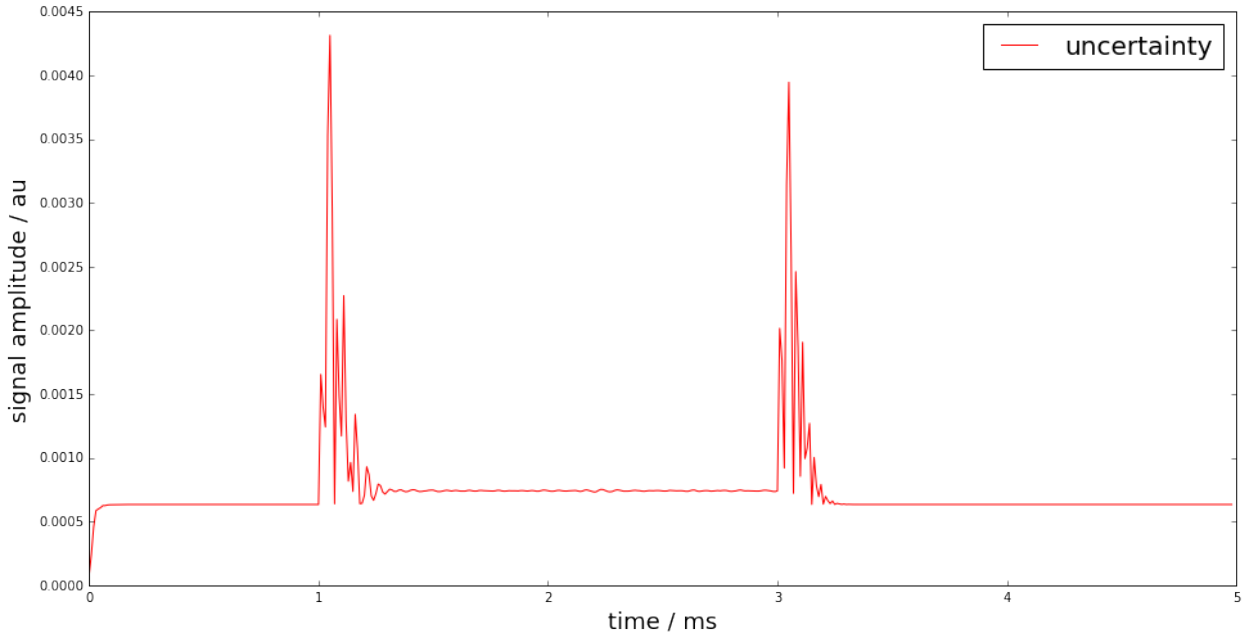

Estimate of the filter output signal and its associated uncertainty

```
y,Uy = IIRuncFilter(x,noise,b,a,Uab)

figure(figsize=(16,8))
plot(time*1e3, x, label="input signal")
plot(time*1e3, y, label="output signal")
legend(fontsize=20)
xlabel('time / ms',fontsize=18)
ylabel('signal amplitude / au',fontsize=18);
```



```
figure(figsize=(16,8))
plot(time*1e3, Uy, "r", label="uncertainty")
legend(fontsize=20)
xlabel('time / ms',fontsize=18)
ylabel('signal amplitude / au',fontsize=18);
```




### Monte-Carlo method for uncertainty propagation

The linearisation-based uncertainty propagation can become unreliable due to the linearisation errors. Therefore, a Monte-Carlo method for digital filters with uncertain coefficients has been proposed in

```
S. Eichstädt, A. Link, P. Harris, and C. Elster,
"Efficient implementation of a Monte Carlo method
for uncertainty evaluation in dynamic measurements,"
Metrologia, vol. 49, no. 3, 2012.
```

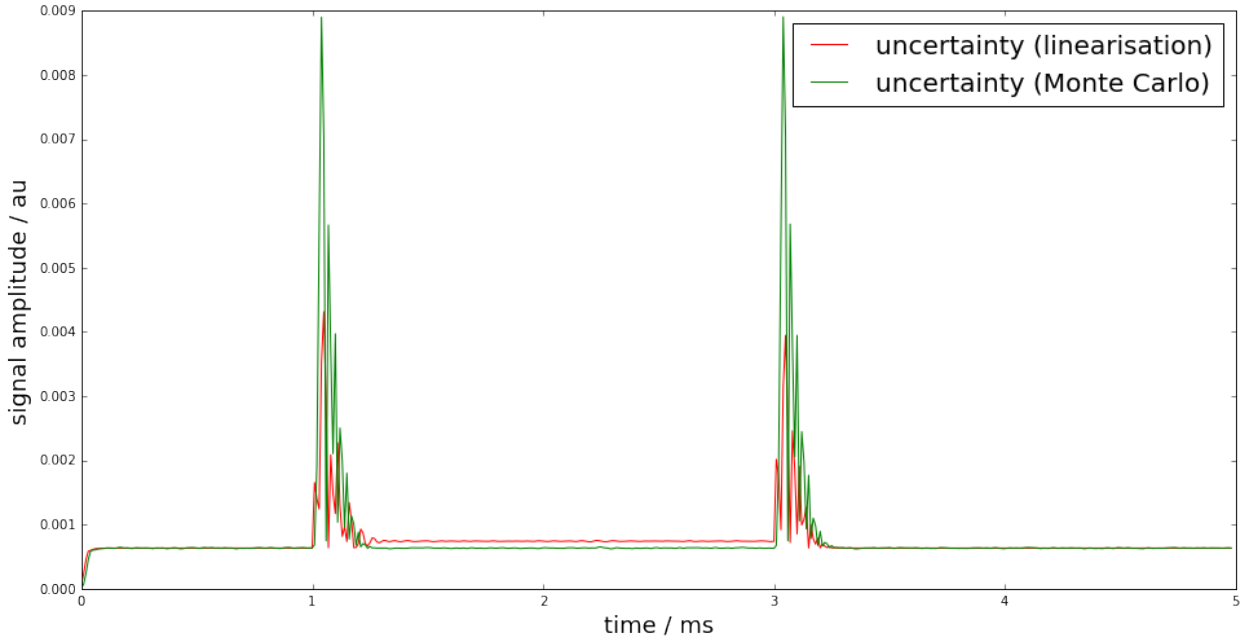
The proposed Monte-Carlo method provides - a memory-efficient implementation of the GUM Monte-Carlo method - online calculation of point-wise uncertainties, estimates and coverage intervals by taking advantage of the sequential character of the filter equation

$$y[k] = \sum_{n=0}^{N_b} b_n x[k-n] - \sum_{m=1}^{N_a} a_m y[k-m]$$

```
yMC,UyMC = SMC(x,noise,b,a,Uab,runs=10000)

figure(figsize=(16,8))
plot(time*1e3, Uy, "r", label="uncertainty (linearisation)")
plot(time*1e3, UyMC, "g", label="uncertainty (Monte Carlo)")
legend(fontsize=20)
xlabel('time / ms',fontsize=18)
ylabel('signal amplitude / au',fontsize=18);
```

```
SMC progress:  0% 10% 20% 30% 40% 50% 60% 70% 80% 90% 100%
```




#### Basic workflow in PyDynamic

#### Using GUM linearization

```
y,Uy = IIRuncFilter(x,noise,b,a,Uab)
```

#### Using sequential GUM Monte Carlo method

```
yMC,UyMC = SMC(x,noise,b,a,Uab,runs=10000)
```

```
SMC progress:  0% 10% 20% 30% 40% 50% 60% 70% 80% 90% 100%
```

```
%pylab inline
colors = [[0.1,0.6,0.5], [0.9,0.2,0.5], [0.9,0.5,0.1]]
```

```
Populating the interactive namespace from numpy and matplotlib
```

### 1.3.3 Deconvolution in the frequency domain (DFT)

```
from PyDynamic.uncertainty.propagate_DFT import GUM_DFT,GUM_iDFT
from PyDynamic.uncertainty.propagate_DFT import DFT_deconv, AmpPhase2DFT
from PyDynamic.uncertainty.propagate_DFT import DFT_multiply
```

```
#%% reference data
ref_file = np.loadtxt("DFTdeconv reference_signal.dat")
time = ref_file[:,0]
ref_data = ref_file[:,1]
Ts = 2e-9
N = len(time)

#%% hydrophone calibration data
calib = np.loadtxt("DFTdeconv calibration.dat")
f = calib[:,0]
FR = calib[:,1]*np.exp(1j*calib[:,3])
Nf = 2*(len(f)-1)

uAmp = calib[:,2]
uPhas= calib[:,4]
UAP = np.r_[uAmp,uPhas*np.pi/180]**2

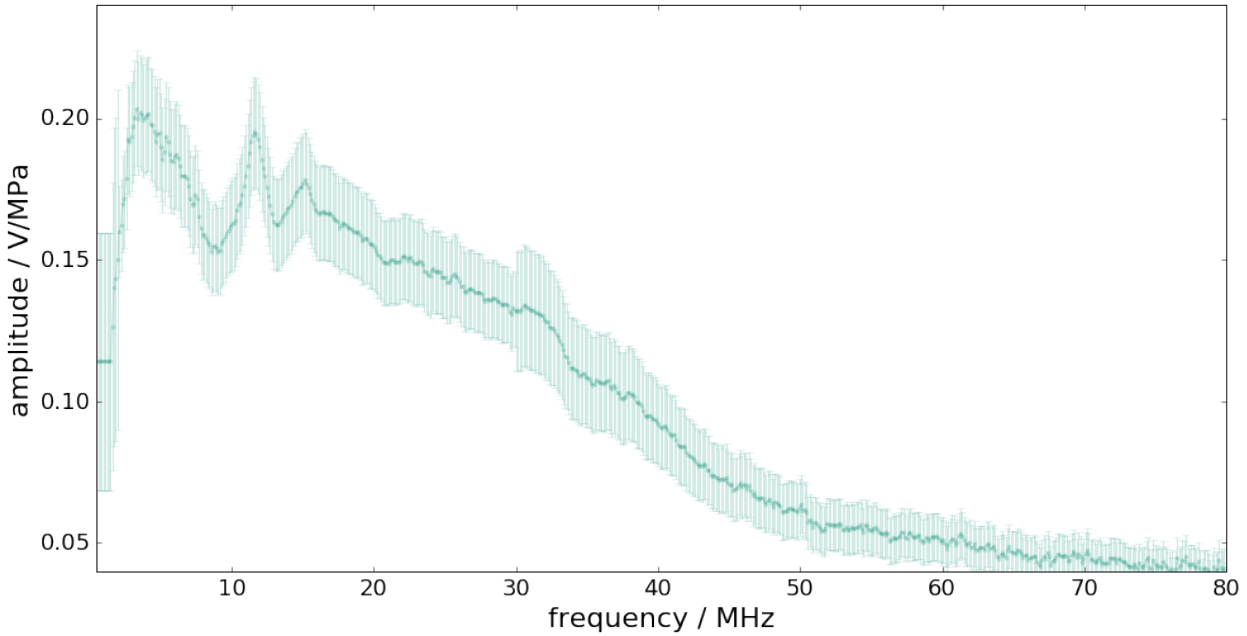
#%% measured hydrophone output signal
meas = np.loadtxt("DFTdeconv measured_signal.dat")
y = meas[:,1]
# assumed noise std
noise_std = 4e-4
Uy = noise_std**2
```

Consider knowledge about the measurement system is available in terms of its frequency response with uncertainties associated with amplitude and phase values.

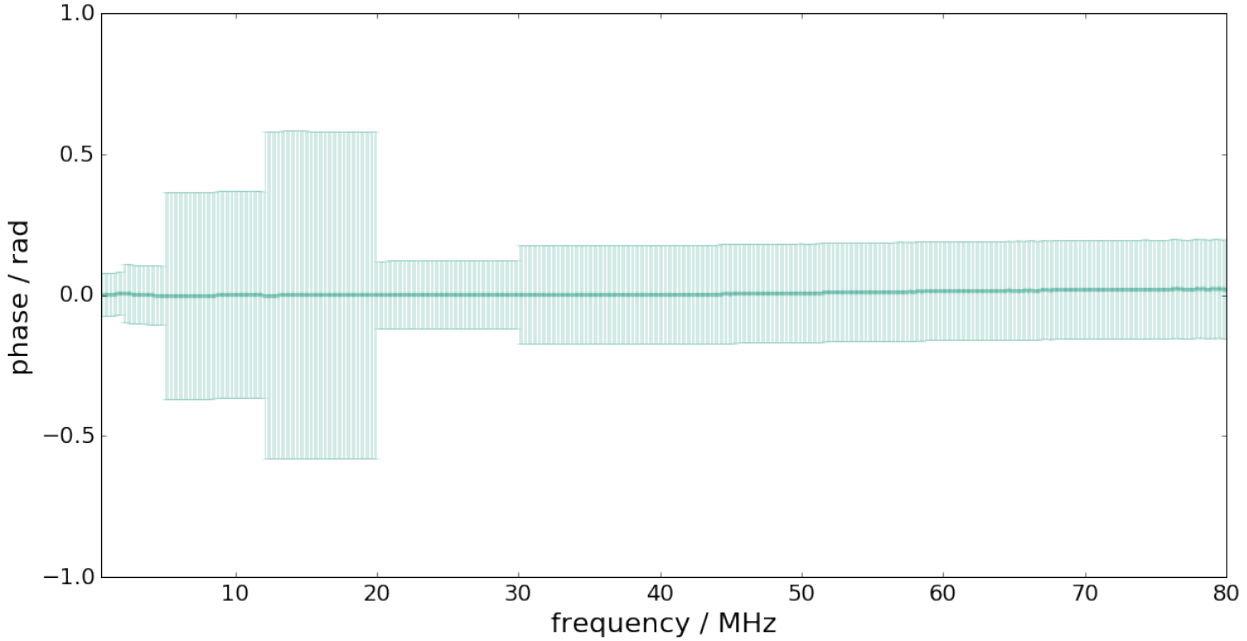
$$\mathbf{H} = (|H(f_1)|, \ldots, \angle H(f_N))$$

$$u_H = (u_{|H(f_1)|}, \ldots, u_{\angle H(f_N)})$$

```
figure(figsize=(16,8))
errorbar(f * 1e-6, abs(FR), 2 * sqrt(UAP[:len(UAP) // 2]), fmt= ".-", alpha=0.2,␣
↪color=colors[0])
xlim(0.5, 80)
ylim(0.04, 0.24)
xlabel("frequency / MHz", fontsize=22); tick_params(which= "both", labelsize=18)
ylabel("amplitude / V/MPa", fontsize=22);
```




```
figure(figsize=(16,8))
errorbar(f * 1e-6, unwrap(angle(FR)) * pi / 180, 2 * UAP[len(UAP) // 2:], fmt= ".-",␣
↪alpha=0.2, color=colors[0])
xlim(0.5, 80)
ylim(-0.2, 0.3)
xlabel("frequency / MHz", fontsize=22); tick_params(which= "both", labelsize=18)
ylim(-1,1)
ylabel("phase / rad", fontsize=22);
```




The measurand is the input signal $\mathbf{x} = (x_1, \ldots, x_M)$ to the measurement system with corresponding measurement model given by

$$y[n] = (h * x)[n] + \varepsilon[n]$$

Input estimation is here to be considered in the Fourier domain.

The estimation model equation is thus given by

$$\hat{x} = \mathcal{F}^{-1}\left(\frac{Y(f)}{H(f)} H_L(f)\right)$$

with - $Y(f)$ the DFT of the measured system output signal - $H_L(f)$ the frequency response of a low-pass filter

Estimation steps

1) DFT of $y$ and propagation of uncertainties to the frequency domain
2) Propagation of uncertainties associated with amplitude and phase of system to corr. real and imaginary parts
3) Division in the frequency domain and propagation of uncertainties
4) Multiplication with low-pass filter and propagation of uncertainties
5) Inverse DFT and propagation of uncertainties to the time domain

### Propagation from time to frequency domain

With the DFT defined as

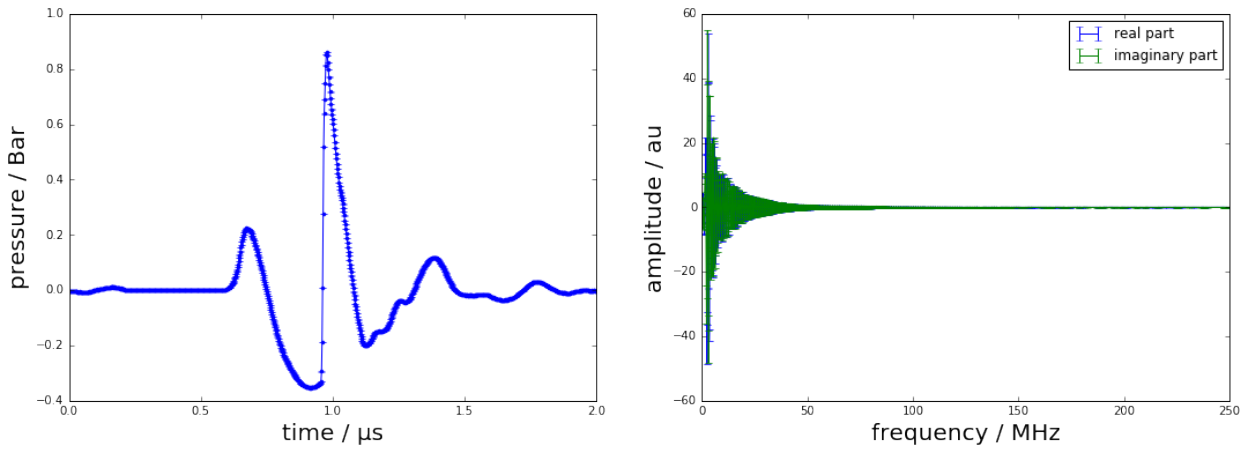
$$Y_k = \sum_{n=0}^{N-1} y_n \exp(-\mathrm{j}k\beta_n)$$

with $\beta_n = 2\pi n/N$, the uncertainty associated with the DFT outcome represented in terms of real and imaginary parts, is given by

$$U_Y = \begin{pmatrix} C_{\cos} U_y C_{\cos}^{\mathsf{T}} & C_{\cos} U_y C_{\sin}^{\mathsf{T}} \\ (C_{\cos} U_y C_{\sin}^{\mathsf{T}})^{\mathsf{T}} & C_{\sin} U_y C_{\sin}^{\mathsf{T}} \end{pmatrix}$$

```
Y,UY = GUM_DFT(y,Uy,N=Nf)
```

```
figure(figsize=(18,6))
subplot(121)
errorbar(time*1e6, y, sqrt(Uy)*ones_like(y),fmt=".-")
xlabel("time / µs",fontsize=20); ylabel("pressure / Bar",fontsize=20)
subplot(122)
errorbar(f*1e-6, Y[:len(f)],sqrt(UY[:len(f)]),label="real part")
errorbar(f*1e-6, Y[len(f):],sqrt(UY[len(f):]),label="imaginary part")
legend()
xlabel("frequency / MHz",fontsize=20); ylabel("amplitude / au",fontsize=20);
```



### Uncertainties for measurement system w.r.t. real and imaginary parts

In practice, the frequency response of the measurement system is characterised in terms of its amplitude and phase values at a certain set of frequencies. GUM uncertainty evaluation, however, requires a representation by real and imaginary parts.

$$H_k = A_k \cos(P_k) + \mathrm{j} A_k \sin(P_k)$$

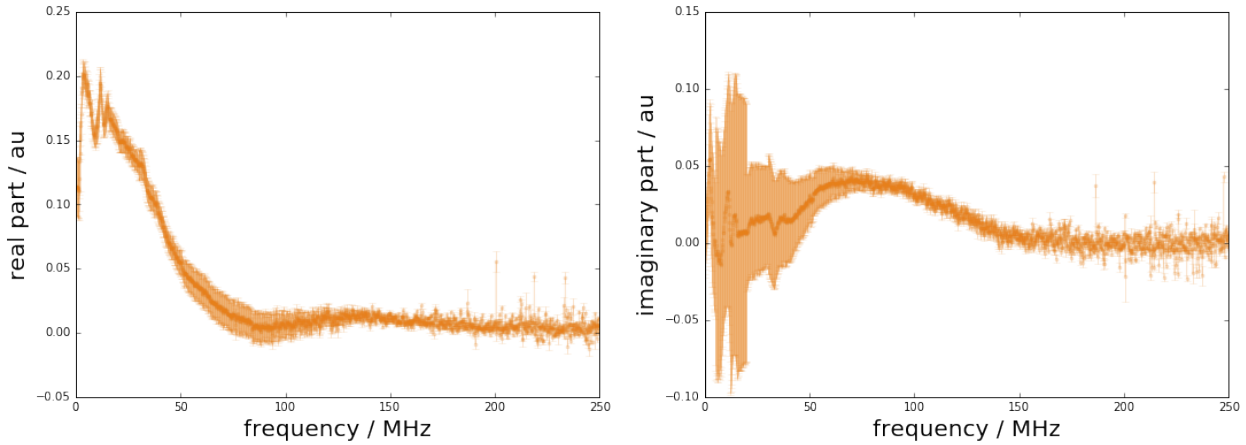
GUM uncertainty propagation

$$C_{RI} = \begin{pmatrix} R_A & R_P \\ I_A & I_P \end{pmatrix}.$$

$$U_H = C_{RI} \begin{pmatrix} U_{AA} & U_{AP} \\ U_{AP}^\mathsf{T} & U_{PP} \end{pmatrix} C_{RI}^\mathsf{T} = \begin{pmatrix} U_{11} & U_{12} \\ U_{21}^\mathsf{T} & U_{22} \end{pmatrix}.$$

```
H, UH = AmpPhase2DFT(np.abs(FR),np.angle(FR),UAP)
```

```
Nf = len(f)
figure(figsize=(18,6))
subplot(121)
errorbar(f*1e-6, H[:Nf], sqrt(diag(UH[:Nf,:Nf])),fmt=".-",color=colors[2],alpha=0.2)
xlabel("frequency / MHz",fontsize=20); ylabel("real part / au",fontsize=20)
subplot(122)
errorbar(f*1e-6, H[Nf:],sqrt(diag(UH[Nf:,Nf:])),fmt=".-",color=colors[2],alpha=0.2)
xlabel("frequency / MHz",fontsize=20); ylabel("imaginary part / au",fontsize=20);
```




### Deconvolution in the frequency domain

The deconvolution problem can be decomposed into a division by the system's frequency response followed by a multiplication by a low-pass filter frequency response.

$$X(f) = \frac{Y(f)}{H(f)} H_L(f)$$

which in real and imaginary part becomes

$$X = \frac{(\Re_Y \Re_H + \Im_Y \Im_H) + \mathrm{j}(-\Re_Y \Im_H + \Im_Y \Re_H)}{\Re_H^2 + \Im_H^2} (\Re_{H_L} + \mathrm{j}\Im_{H_L})$$

Sensitivities for division part

$$R_{RY} = \frac{\partial \Re_X}{\partial \Re_Y} = \frac{\Re_H}{\Re_H^2 + \Im_H^2} \tag{1.6}$$

$$R_{IY} = \frac{\partial \Re_X}{\partial \Im_Y} = \frac{\Im_H}{\Re_H^2 + \Im_H^2} \tag{1.7}$$

$$R_{RH} = \frac{\partial \Re_X}{\partial \Re_H} = \frac{-\Re_Y \Re_H^2 + \Re_Y \Im_H^2 - 2\Im_Y \Im_H \Re_H}{(\Re_H^2 + \Im_H^2)^2} \tag{1.8}$$

$$R_{IH} = \frac{\partial \Re_X}{\partial \Im_H} = \frac{\Im_Y \Re_H^2 - \Im_Y \Im_H^2 - 2\Re_Y \Re_H \Im_H}{(\Re_H^2 + \Im_H^2)^2} \tag{1.9}$$

$$I_{RY} = \frac{\partial \Im_X}{\partial \Re_Y} = \frac{-\Im_H}{\Re_H^2 + \Im_H^2} \tag{1.10}$$

$$I_{IY} = \frac{\partial \Im_X}{\partial \Im_Y} = \frac{\Re_H}{\Re_H^2 + \Im_H^2} \tag{1.11}$$

$$I_{RH} = \frac{\partial \Im_X}{\partial \Re_H} = \frac{-\Im_Y \Re_H^2 + \Im_Y \Im_H^2 + 2\Re_Y \Im_H \Re_H}{(\Re_H^2 + \Im_H^2)^2} \tag{1.12}$$

$$I_{IH} = \frac{\partial \Im_X}{\partial \Im_H} = \frac{-\Re_Y \Re_H^2 + \Re_Y \Im_H^2 - 2\Im_Y \Re_H \Im_H}{(\Re_H^2 + \Im_H^2)^2} \tag{1.13}$$

Uncertainty blocks for multiplication part

$$U_{XRR} = \Re_{H_L} U_{ARR} \Re_{H_L} - \Im_{H_L} U_{ARI}^T \Re_{H_L} - \Re_{H_L} U_{ARI} \Im_{H_L} + \Im_{H_L} U_{AII} \Im_{H_L} \tag{1.14}$$

$$U_{XRI} = \Re_{H_L} U_{ARR} \Im_{H_L} - \Im_{H_L} U_{ARI}^T \Im_{H_L} + \Re_{H_L} U_{ARI} \Re_{H_L} - \Im_{H_L} U_{AII} \Re_{H_L} \tag{1.15}$$

$$U_{XIR} = U_{YRI}^T \tag{1.16}$$

$$U_{XII} = \Im_{H_L} U_{ARR} \Im_{H_L} + \Re_{H_L} U_{ARI}^T \Im_{H_L} + \Im_{H_L} U_{ARI} \Re_{H_L} + \Re_{H_L} U_{AII} \Re_{H_L} \tag{1.17}$$

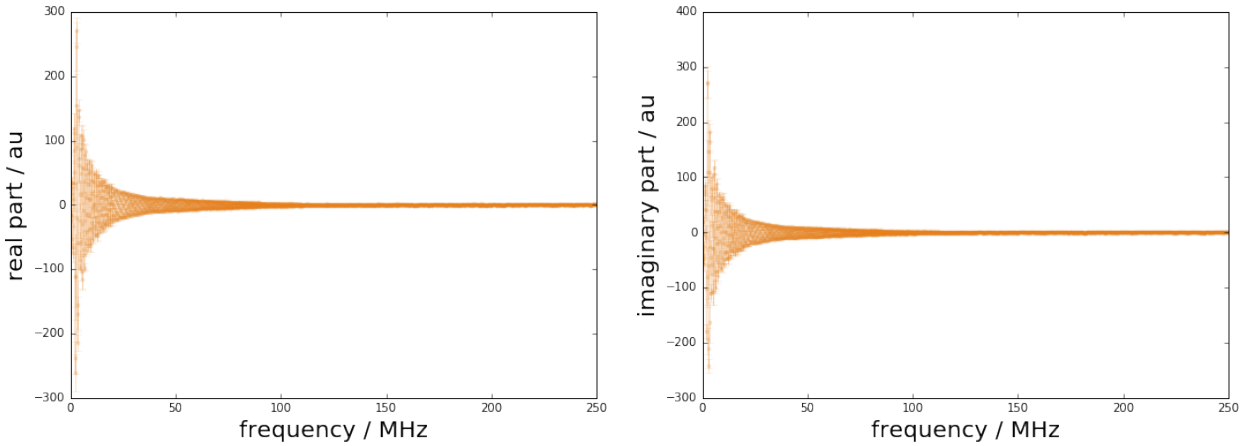
```
# low-pass filter for deconvolution
def lowpass(f,fcut=80e6):
    return 1/(1+1j*f/fcut)**2

HLc = lowpass(f)
HL = np.r_[np.real(HLc), np.imag(HLc)]
```

```
XH,UXH = DFT_deconv(H,Y,UH,UY)

XH, UXH = DFT_multiply(XH, UXH, HL)
```

```
figure(figsize=(18,6))
subplot(121)
errorbar(f*1e-6, XH[:Nf], sqrt(diag(UXH[:Nf,:Nf])),fmt=".-",color=colors[2],alpha=0.2)
xlabel("frequency / MHz",fontsize=20); ylabel("real part / au",fontsize=20)
subplot(122)
errorbar(f*1e-6, XH[Nf:],sqrt(diag(UXH[Nf:,Nf:])),fmt=".-",color=colors[2],alpha=0.2)
xlabel("frequency / MHz",fontsize=20); ylabel("imaginary part / au",fontsize=20);
```




### Propagation from frequency to time domain

The inverse DFT equation is given by

$$X_n = \frac{1}{N}\sum_{k=0}^{N-1}(\Re_k \cos(k\beta_n) - \Im_k \sin(k\beta_n))$$

The sensitivities for the GUM propagation of uncertainties are then

$$\frac{\partial X_n}{\partial \Re_k} = \frac{1}{N} \quad \text{for } k = 0 \tag{1.18}$$

$$\frac{\partial X_n}{\partial \Re_k} = \frac{2}{N}\cos(k\beta_n) \quad \text{for } k = 1,\ldots,N/2-1 \tag{1.19}$$

$$\frac{\partial X_n}{\partial \Im_k} = 0 \quad \text{for } k = 0 \tag{1.20}$$

$$\frac{\partial X_n}{\partial \Im_k} = -\frac{2}{N}\sin(k\beta_n) \quad \text{for } k = 1,\ldots,N/2-1. \tag{1.21}$$

GUM uncertainty propagation for the inverse DFT

$$C_F U_F C_F^{\mathsf{T}} = \left(\widetilde{C}_{\cos}, \widetilde{C}_{\sin}\right)\begin{pmatrix} U_{RR} & U_{RI} \\ U_{IR} & U_{II}\end{pmatrix}\begin{pmatrix} \widetilde{C}_{\cos}^{\mathsf{T}} \\ \widetilde{C}_{\sin}^{\mathsf{T}}\end{pmatrix} \tag{1.22}$$

$$= \widetilde{C}_{\cos} U_{RR} \widetilde{C}_{\cos}^{\mathsf{T}} + 2\widetilde{C}_{\cos} U_{RI} \widetilde{C}_{\sin}^{\mathsf{T}} + \widetilde{C}_{\sin} U_{II} \widetilde{C}_{\sin}^{\mathsf{T}} \tag{1.23}$$

```
xh,Uxh = GUM_iDFT(XH,UXH,Nx=N)
```

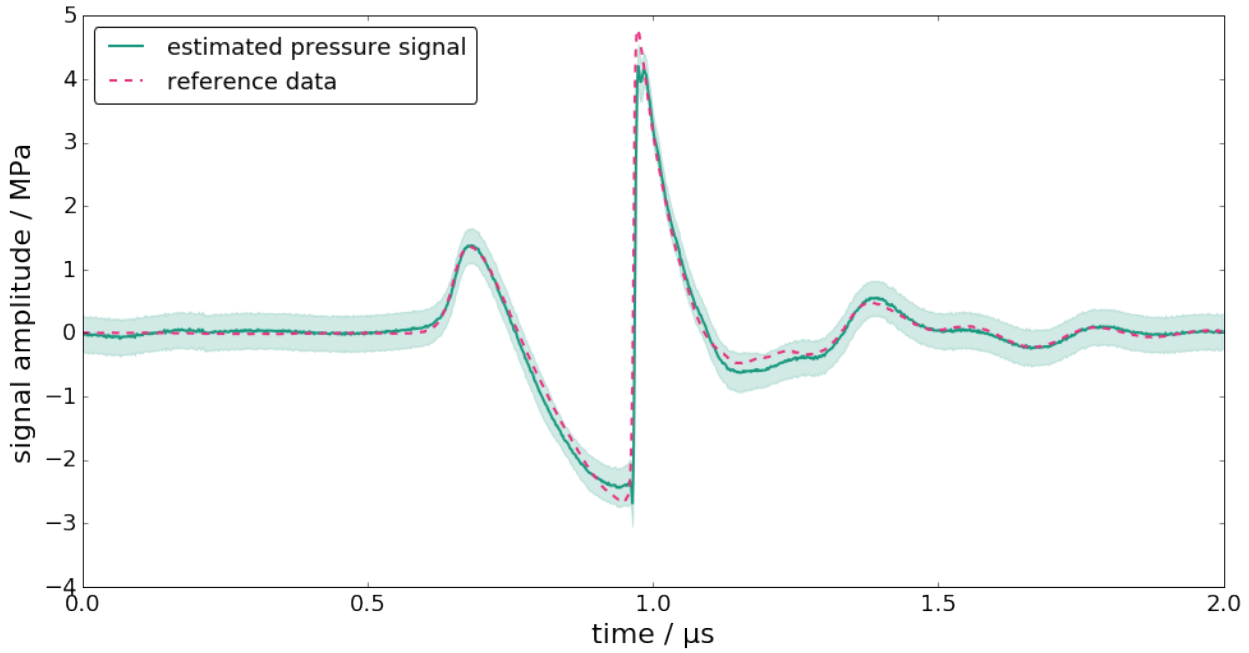
```
ux = np.sqrt(np.diag(Uxh))

figure(figsize=(16,8))
plot(time*1e6,xh,label="estimated pressure signal",linewidth=2,color=colors[0])
plot(time * 1e6, ref_data, "--", label= "reference data", linewidth=2,␣
↪color=colors[1])
fill_between(time * 1e6, xh + 2 * ux, xh - 2 * ux, alpha=0.2, color=colors[0])
```

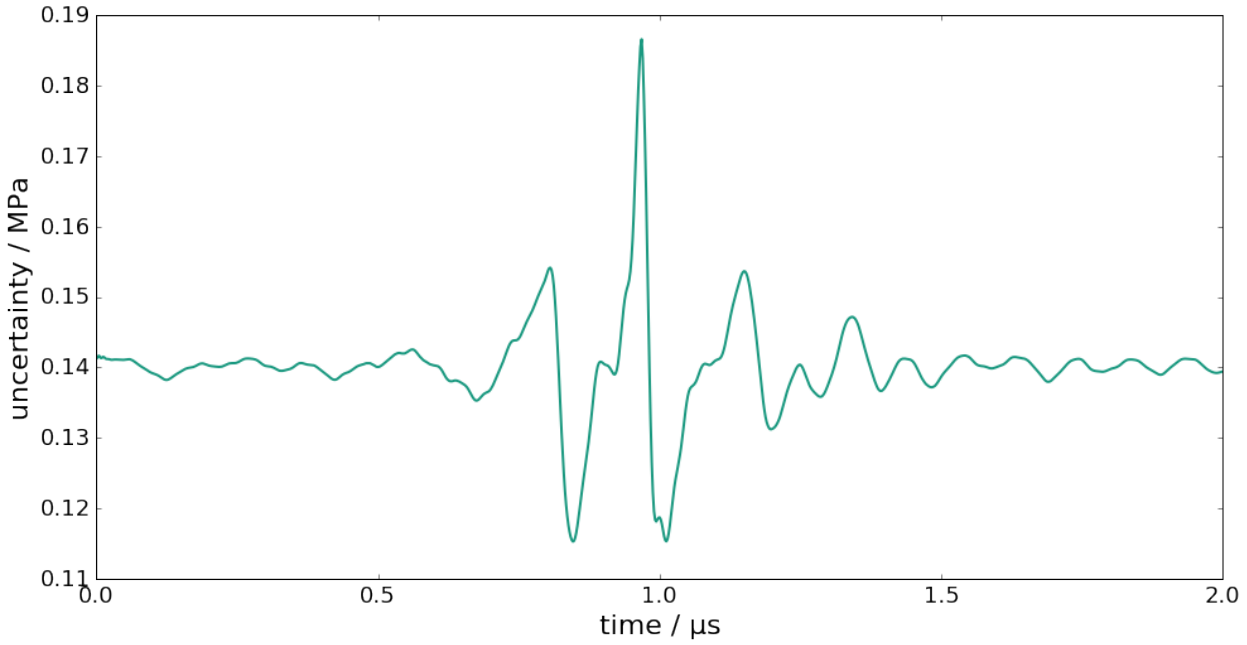
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```
xlabel("time / µs", fontsize=22)
ylabel("signal amplitude / MPa", fontsize=22)
tick_params(which= "major", labelsize=18)
legend(loc= "upper left", fontsize=18, fancybox=True)
xlim(0, 2);
```




```
figure(figsize=(16,8))
plot(time * 1e6, ux, label= "uncertainty", linewidth=2, color=colors[0])
xlabel("time / µs", fontsize=22)
ylabel("uncertainty / MPa", fontsize=22)
tick_params(which= "major", labelsize=18)
xlim(0,2);
```




### Summary of PyDynamic workflow for deconvolution in DFT domain

```
Y,UY = GUM_DFT(y,Uy,N=Nf)

H, UH = AmpPhase2DFT(A, P, UAP)

XH,UXH = DFT_deconv(H,Y,UH,UY)

XH, UXH = DFT_multiply(XH, UXH, HL)
```

CHAPTER 2

# Evaluation of uncertainties

The evaluation of uncertainties is a fundamental part of the measurement analysis in metrology. The analysis of dynamic measurements typically involves methods from signal processing, such as digital filtering or application of the discrete Fourier transform (DFT). For most tasks, methods are readily available, for instance, as part of `scipy.signals`. This module of PyDynamic provides the corresponding methods for the evaluation of uncertainties.

## 2.1 Uncertainty evaluation for the DFT

The *PyDynamic.uncertainty.propagate_DFT* module implements methods for the propagation of uncertainties in the application of the DFT, inverse DFT, deconvolution and multiplication in the frequency domain, transformation from amplitude and phase to real and imaginary parts and vice versa.

**The correspoding scientific publications is** S. Eichstädt und V. Wilkens GUM2DFT — a software tool for uncertainty evaluation of transient signals in the frequency domain. *Measurement Science and Technology*, 27(5), 055001, 2016. [DOI: 10.1088/0957-0233/27/5/055001]

This module contains the following functions: * GUM_DFT: Calculation of the DFT of the time domain signal x and propagation of the squared uncertainty Ux

associated with the time domain sequence x to the real and imaginary parts of the DFT of x

- **GUM_iDFT: GUM propagation of the squared uncertainty UF associated with the DFT values F through the** inverse DFT
- GUM_DFTfreq: Return the Discrete Fourier Transform sample frequencies
- **DFT_transferfunction: Calculation of the transfer function H = Y/X in the frequency domain with X being the Fourier tra** of the system's input signal and Y that of the output signal
- DFT_deconv: Deconvolution in the frequency domain
- DFT_multiply: Multiplication in the frequency domain
- AmpPhase2DFT: Transformation from magnitude and phase to real and imaginary parts
- DFT2AmpPhase: Transformation from real and imaginary parts to magnitude and phase

- AmpPhase2Time: Transformation from amplitude and phase to time domain
- Time2AmpPhase: Transformation from time domain to amplitude and phase

PyDynamic.uncertainty.propagate_DFT.**GUM_DFT**(*x*, *Ux*, *N=None*, *window=None*, *CxCos=None*, *CxSin=None*, *returnC=False*, *mask=None*)

Calculation of the DFT of the time domain signal x and propagation of the squared uncertainty Ux associated with the time domain sequence x to the real and imaginary parts of the DFT of x.

**Parameters**

- **x** (*numpy.ndarray of shape (M,)*) – vector of time domain signal values
- **Ux** (*numpy.ndarray*) – covariance matrix associated with x, shape (M,M) or noise variance as float
- **N** (*int, optional*) – length of time domain signal for DFT; N>=len(x)
- **window** (*numpy.ndarray, optional of shape (M,)*) – vector of the time domain window values
- **CxCos** (*numpy.ndarray, optional*) – cosine part of sensitivity matrix
- **CxSin** (*numpy.ndarray, optional*) – sine part of sensitivity matrix
- **returnC** (*bool, optional*) – if true, return sensitivity matrix blocks for later use
- **mask** (*ndarray of dtype bool*) – calculate DFT values and uncertainties only at those frequencies where mask is *True*

**Returns**

- **F** (*numpy.ndarray*) – vector of complex valued DFT values or of its real and imaginary parts
- **UF** (*numpy.ndarray*) – covariance matrix associated with real and imaginary part of F

### References


- Eichstädt and Wilkens [Eichst2016]


PyDynamic.uncertainty.propagate_DFT.**GUM_iDFT**(*F*, *UF*, *Nx=None*, *Cc=None*, *Cs=None*, *returnC=False*)

GUM propagation of the squared uncertainty UF associated with the DFT values F through the inverse DFT

The matrix UF is assumed to be for real and imaginary part with blocks: UF = [[u(R,R), u(R,I)],[u(I,R),u(I,I)]] and real and imaginary part obtained from calling rfft (DFT for real-valued signal)

**Parameters**

- **F** (*np.ndarray of shape (2M,)*) – vector of real and imaginary parts of a DFT result
- **UF** (*np.ndarray of shape (2M,2M)*) – covariance matrix associated with real and imaginary parts of F
- **Nx** (*int, optional*) – number of samples of iDFT result
- **Cc** (*np.ndarray, optional*) – cosine part of sensitivities
- **Cs** (*np.ndarray, optional*) – sine part of sensitivities
- **returnC** (*if true, return sensitivity matrix blocks*) –

**Returns**

- **x** (*np.ndarry*) – vector of time domain signal values
- **Ux** (*np.ndarray*) – covariance matrix associated with x

### References

- Eichstädt and Wilkens [Eichst2016]

PyDynamic.uncertainty.propagate_DFT.**GUM_DFTfreq**(*N*, *dt=1*)
Return the Discrete Fourier Transform sample frequencies

**Parameters**

- **N** (*int*) – window length
- **dt** (*float*) – sample spacing (inverse of sampling rate)

**Returns** **f** – Array of length `n//2 + 1` containing the sample frequencies

**Return type** ndarray

PyDynamic.uncertainty.propagate_DFT.**DFT_transferfunction**(*X*, *Y*, *UX*, *UY*)
Calculation of the transfer function H = Y/X in the frequency domain with X being the Fourier transform of the system's input signal and Y that of the output signal.

**Parameters**

- **X** (*np.ndarray*) – real and imaginary parts of the system's input signal
- **Y** (*np.ndarray*) – real and imaginary parts of the system's output signal
- **UX** (*np.ndarray*) – covariance matrix associated with X
- **UY** (*np.ndarray*) – covariance matrix associated with Y

**Returns**

- **H** (*np.ndarray*) – real and imaginary parts of the system's frequency response
- **UH** (*np.ndarray*) – covariance matrix associated with H

This function uses *DFT_deconv*.

PyDynamic.uncertainty.propagate_DFT.**DFT_deconv**(*H*, *Y*, *UH*, *UY*)
Deconvolution in the frequency domain

GUM propagation of uncertainties for the deconvolution X = Y/H with Y and H being the Fourier transform of the measured signal and of the system's impulse response, respectively. This function returns the covariance matrix as a tuple of blocks if too large for complete storage in memory.

**Parameters**

- **H** (*np.ndarray of shape (2M,)*) – real and imaginary parts of frequency response values (N an even integer)
- **Y** (*np.ndarray of shape (2M,)*) – real and imaginary parts of DFT values
- **UH** (*np.ndarray of shape (2M,2M)*) – covariance matrix associated with H
- **UY** (*np.ndarray of shape (2M,2M)*) – covariance matrix associated with Y

**Returns**

- **X** (*np.ndarray of shape (2M,)*) – real and imaginary parts of DFT values of deconv result

- **UX** (*np.ndarray of shape (2M,2M)*) – covariance matrix associated with real and imaginary part of X

### References


- Eichstädt and Wilkens [Eichst2016]


PyDynamic.uncertainty.propagate_DFT.**DFT_multiply**(*Y*, *F*, *UY*, *UF=None*)

Multiplication in the frequency domain

GUM uncertainty propagation for multiplication in the frequency domain, where the second factor F may have an associated uncertainty. This method can be used, for instance, for the application of a low-pass filter in the frequency domain or the application of deconvolution as a multiplication with an inverse of known uncertainty.

**Parameters**

- **Y** (*np.ndarray of shape (2M,)*) – real and imaginary parts of the first factor
- **F** (*np.ndarray of shape (2M,)*) – real and imaginary parts of the second factor
- **UY** (*np.ndarray either shape (2M,) or shape (2M,2M)*) – covariance matrix or squared uncertainty associated with Y
- **UF** (*np.ndarray of shape (2M,2M)*) – covariance matrix associated with F (optional), default is None

**Returns**

- **YF** (*np.ndarray of shape (2M,)*) – the product of Y and F
- **UYF** (*np.ndarray of shape (2M,2M)*) – the uncertainty associated with YF

PyDynamic.uncertainty.propagate_DFT.**AmpPhase2DFT**(*A*, *P*, *UAP*, *keep_sparse=False*)

Transformation from magnitude and phase to real and imaginary parts

Calculate the vector F=[real,imag] and propagate the covariance matrix UAP associated with [A, P]

**Parameters**

- **A** (*np.ndarray of shape (N,)*) – vector of magnitude values
- **P** (*np.ndarray of shape (N,)*) – vector of phase values (in radians)
- **UAP** (*np.ndarray of shape (2N,2N)*) – covariance matrix associated with (A,P) or vector of squared standard uncertainties [u^2(A),u^2(P)]
- **keep_sparse** (*bool, optional*) – whether to transform sparse matrix to numpy array or not

**Returns**

- **F** (*np.ndarray*) – vector of real and imaginary parts of DFT result
- **UF** (*np.ndarray*) – covariance matrix associated with F

PyDynamic.uncertainty.propagate_DFT.**DFT2AmpPhase**(*F*, *UF*, *keep_sparse=False*, *tol=1.0*, *return_type='separate'*)

Transformation from real and imaginary parts to magnitude and phase

Calculate the matrix U_AP = [[U1,U2],[U2^T,U3]] associated with magnitude and phase of the vector F=[real,imag] with associated covariance matrix U_F=[[URR,URI],[URI^T,UII]]

**Parameters**

- **F** (*np.ndarray of shape (2M,)*) – vector of real and imaginary parts of a DFT result
- **UF** (*np.ndarray of shape (2M,2M)*) – covariance matrix associated with F
- **keep_sparse** (*bool, optional*) – if true then UAP will be sparse if UF is one-dimensional
- **tol** (*float, optional*) – lower bound for A/uF below which a warning will be issued concerning unreliable results
- **return_type** (*str, optional*) – If "separate" then magnitude and phase are returned as separate arrays. Otherwise the array [A, P] is returned

**Returns**

- If *return_type* is *separate* –

  **A: np.ndarray** vector of magnitude values

  **P: np.ndarray** vector of phase values in radians, in the range [-pi, pi]

  **UAP: np.ndarray** covariance matrix associated with (A,P)

- *Otherwise* –

  **AP: np.ndarray** vector of magnitude and phase values

  **UAP: np.ndarray** covariance matrix associated with AP

PyDynamic.uncertainty.propagate_DFT.**AmpPhase2Time**(*A*, *P*, *UAP*)

Transformation from amplitude and phase to time domain

GUM propagation of covariance matrix UAP associated with DFT amplitude A and phase P to the result of the inverse DFT. Uncertainty UAP is assumed to be given for amplitude and phase with blocks: UAP = [[u(A,A), u(A,P)],[u(P,A),u(P,P)]]

**Parameters**

- **A** (*np.ndarray of shape (N,)*) – vector of amplitude values
- **P** (*np.ndarray of shape (N,)*) – vector of phase values (in rad)
- **UAP** (*np.ndarray of shape (2N,2N)*) – covariance matrix associated with [A,P]

**Returns**

- **x** (*np.ndarray*) – vector of time domain values
- **Ux** (*np.ndarray*) – covariance matrix associated with x

PyDynamic.uncertainty.propagate_DFT.**Time2AmpPhase**(*x*, *Ux*)

Transformation from time domain to amplitude and phase

**Parameters**

- **x** (*np.ndarray of shape (N,)*) – time domain signal
- **Ux** (*np.ndarray of shape (N,N)*) – squared uncertainty associated with x

**Returns**

- **A** (*np.ndarray*) – amplitude values
- **P** (*np.ndarray*) – phase values
- **UAP** (*np.ndarray*) – covariance matrix associated with [A,P]

PyDynamic.uncertainty.propagate_DFT.**Time2AmpPhase_multi**(*x*, *Ux*, *selector=None*)

Transformation from time domain to amplitude and phase for a set of M signals of the same type

**Parameters**

- **x** (*np.ndarray of shape (M,N)*) – M time domain signals of length N
- **Ux** (*np.ndarray of shape (M,)*) – squared standard deviations representing noise variances of the signals x
- **selector** (*np.ndarray of shape (L,), optional*) – indices of amplitude and phase values that should be returned; default is 0:N-1

**Returns**

- **A** (*np.ndarray of shape (M,N)*) – amplitude values
- **P** (*np.ndarray of shape (M,N)*) – phase values
- **UAP** (*np.ndarray of shape (M, 3N)*) – diagonals of the covariance matrices: [diag(UPP), diag(UAA), diag(UPA)]

## 2.2 Uncertainty evaluation for digital filtering

This module contains functions for the propagation of uncertainties through the application of a digital filter using the GUM approach.

This modules contains the following functions: * FIRuncFilter: Uncertainty propagation for signal y and uncertain FIR filter theta * IIRuncFilter: Uncertainty propagation for the signal x and the uncertain IIR filter (b,a)

# Note: The Elster-Link paper for FIR filters assumes that the autocovariance is known and that noise is stationary!

PyDynamic.uncertainty.propagate_filter.**FIRuncFilter**(*y*, *sigma_noise*, *theta*, *Utheta=None*, *shift=0*, *blow=None*)

Uncertainty propagation for signal y and uncertain FIR filter theta

**Parameters**

- **y** (*np.ndarray*) – filter input signal
- **sigma_noise** (*float or np.ndarray*) – when float then standard deviation of white noise in y; when ndarray then point-wise standard uncertainties
- **theta** (*np.ndarray*) – FIR filter coefficients
- **Utheta** (*np.ndarray*) – covariance matrix associated with theta
- **shift** (*int*) – time delay of filter output signal (in samples)
- **blow** (*np.ndarray*) – optional FIR low-pass filter

**Returns**

- **x** (*np.ndarray*) – FIR filter output signal
- **ux** (*np.ndarray*) – point-wise uncertainties associated with x

### References


- Elster and Link 2008 [Elster2008]

**See also:**

*PyDynamic.deconvolution.fit_filter*

PyDynamic.uncertainty.propagate_filter.**IIRuncFilter**(*x*, *noise*, *b*, *a*, *Uab*)

Uncertainty propagation for the signal x and the uncertain IIR filter (b,a)

**Parameters**

- **x** (*np.ndarray*) – filter input signal
- **noise** (*float*) – signal noise standard deviation
- **b** (*np.ndarray*) – filter numerator coefficients
- **a** (*np.ndarray*) – filter denominator coefficients
- **Uab** (*np.ndarray*) – covariance matrix for (a[1:],b)

**Returns**

- **y** (*np.ndarray*) – filter output signal
- **Uy** (*np.ndarray*) – uncertainty associated with y

### References


- Link and Elster [Link2009]


## 2.3 Monte Carlo methods for digital filtering

The propagation of uncertainties via the FIR and IIR formulae alone does not enable the derivation of credible intervals, because the underlying distribution remains unknown. The GUM-S2 Monte Carlo method provides a reference method for the calculation of uncertainties for such cases.

This module implements Monte Carlo methods for the propagation of uncertainties for digital filtering.

This module contains the following functions:

- *MC*: Standard Monte Carlo method for application of digital filter
- *SMC*: Sequential Monte Carlo method with reduced computer memory requirements
- *UMC*: Update Monte Carlo method for application of digital filters with reduced computer memory requirements
- *UMC_generic*: Update Monte Carlo method with reduced computer memory requirements

PyDynamic.uncertainty.propagate_MonteCarlo.**MC**(*x*, *Ux*, *b*, *a*, *Uab*, *runs=1000*, *blow=None*, *alow=None*, *return_samples=False*, *shift=0*, *verbose=True*)

Standard Monte Carlo method

Monte Carlo based propagation of uncertainties for a digital filter (b,a) with uncertainty matrix $U_\theta$ for $\theta = (a_1, \ldots, a_{N_a}, b_0, \ldots, b_{N_b})^T$

**Parameters**

- **x** (*np.ndarray*) – filter input signal
- **Ux** (*float or np.ndarray*) – standard deviation of signal noise (float), point-wise standard uncertainties or covariance matrix associated with x
- **b** (*np.ndarray*) – filter numerator coefficients

- **a** (*np.ndarray*) – filter denominator coefficients
- **Uab** (*np.ndarray*) – uncertainty matrix $U_\theta$
- **runs** (*int, optional*) – number of Monte Carlo runs
- **return_samples** (*bool, optional*) – whether samples or mean and std are returned

If `return_samples` is `False`, the method returns:

**Returns**

- **y** (*np.ndarray*) – filter output signal
- **Uy** (*np.ndarray*) – uncertainty associated with

Otherwise the method returns:

**Returns** **Y** – array of Monte Carlo results

**Return type** np.ndarray

### References

- Eichstädt, Link, Harris and Elster [Eichst2012]

PyDynamic.uncertainty.propagate_MonteCarlo.**SMC**(*x*, *noise_std*, *b*, *a*, *Uab=None*, *runs=1000*, *Perc=None*, *blow=None*, *alow=None*, *shift=0*, *return_samples=False*, *phi=None*, *theta=None*, *Delta=0.0*)

Sequential Monte Carlo method

Sequential Monte Carlo propagation for a digital filter (b,a) with uncertainty matrix $U_\theta$ for $\theta = (a_1, \ldots, a_{N_a}, b_0, \ldots, b_{N_b})^T$

**Parameters**

- **x** (*np.ndarray*) – filter input signal
- **noise_std** (*float*) – standard deviation of signal noise
- **b** (*np.ndarray*) – filter numerator coefficients
- **a** (*np.ndarray*) – filter denominator coefficients
- **Uab** (*np.ndarray*) – uncertainty matrix $U_\theta$
- **runs** (*int, optional*) – number of Monte Carlo runs
- **Perc** (*list, optional*) – list of percentiles for quantile calculation
- **blow** (*np.ndarray*) – optional low-pass filter numerator coefficients
- **alow** (*np.ndarray*) – optional low-pass filter denominator coefficients
- **shift** (*int*) – integer for time delay of output signals
- **return_samples** (*bool, otpional*) – whether to return y and Uy or the matrix Y of MC results
- **theta** (*phi,*) – parameters for AR(MA) noise model $\epsilon(n) = \sum_k \phi_k \epsilon(n-k) + \sum_k \theta_k w(n-k) + w(n)$ with $w(n) \sim N(0, noise_std^2)$
- **Delta** (*float, optional*) – upper bound on systematic error of the filter

If `return_samples` is `False`, the method returns:

**Returns**

- **y** (*np.ndarray*) – filter output signal (Monte Carlo mean)
- **Uy** (*np.ndarray*) – uncertainties associated with y (Monte Carlo point-wise std)
- **Quant** (*np.ndarray*) – quantiles corresponding to percentiles `Perc` (if not `None`)

Otherwise the method returns:

**Returns** **Y** – array of all Monte Carlo results

**Return type** np.ndarray

### References


- Eichstädt, Link, Harris, Elster [Eichst2012]


`PyDynamic.uncertainty.propagate_MonteCarlo.`**`UMC`**(*x*, *b*, *a*, *Uab*, *runs=1000*, *blocksize=8*, *blow=1.0*, *alow=1.0*, *phi=0.0*, *theta=0.0*, *sigma=1*, *Delta=0.0*, *runs_init=100*, *nbins=1000*, *credible_interval=0.95*)

Batch Monte Carlo for filtering using update formulae for mean, variance and (approximated) histogram. This is a wrapper for the UMC_generic function, specialised on filters

**Parameters**

- **x** (*np.ndarray, shape (nx, )*) – filter input signal
- **b** (*np.ndarray, shape (nbb, )*) – filter numerator coefficients
- **a** (*np.ndarray, shape (naa, )*) – filter denominator coefficients, normalization (a[0] == 1.0) is assumed
- **Uab** (*np.ndarray, shape (naa + nbb - 1, )*) – uncertainty matrix $U_\theta$
- **runs** (*int, optional*) – number of Monte Carlo runs
- **blocksize** (*int, optional*) – how many samples should be evaluated for at a time
- **blow** (*float or np.ndarray, optional*) – filter coefficients of optional low pass filter
- **alow** (*float or np.ndarray, optional*) – filter coefficients of optional low pass filter
- **phi** (*np.ndarray, optional,*) – see misc.noise.ARMA noise model
- **theta** (*np.ndarray, optional*) – see misc.noise.ARMA noise model
- **sigma** (*float, optional*) – see misc.noise.ARMA noise model
- **Delta** (*float, optional*) – upper bound of systematic correction due to regularisation (assume uniform distribution)
- **runs_init** (*int, optional*) – how many samples to evaluate to form initial guess about limits
- **nbins** (*int, list of int, optional*) – number of bins for histogram
- **credible_interval** (*float, optional*) – must be in [0,1] central credible interval size

By default, phi, theta, sigma are chosen such, that N(0,1)-noise is added to the input signal.

**Returns**

- **y** (*np.ndarray*) – filter output signal
- **Uy** (*np.ndarray*) – uncertainty associated with
- **y_cred_low** (*np.ndarray*) – lower boundary of credible interval
- **y_cred_high** (*np.ndarray*) – upper boundary of credible interval
- **happr** (*dict*) – dictionary keys: given nbin dictionary values: bin-edges val["bin-edges"], bin-counts val["bin-counts"]

### References

- Eichstädt, Link, Harris, Elster [Eichst2012]
- ported to python in 2019-08 from matlab-version of Sascha Eichstaedt (PTB) from 2011-10-12
- copyright on updating formulae parts is by Peter Harris (NPL)

PyDynamic.uncertainty.propagate_MonteCarlo.**UMC_generic**(*draw_samples*, *evaluate*, *runs=100*, *blocksize=8*, *runs_init=10*, *nbins=100*, *return_samples=False*, *n_cpu=4*)

Generic Batch Monte Carlo using update formulae for mean, variance and (approximated) histogram. Assumes that the input and output of evaluate are numeric vectors (but not necessarily of same dimension).

**Parameters**

- **draw_samples** (*function(int nDraws)*) – function that draws nDraws from a given distribution / population
- **evaluate** (*function(sample)*) – function that evaluates a sample and returns the result
- **runs** (*int, optional*) – number of Monte Carlo runs
- **blocksize** (*int, optional*) – how many samples should be evaluated for at a time
- **runs_init** (*int, optional*) – how many samples to evaluate to form initial guess about limits
- **nbins** (*int, list of int, optional*) – number of bins for histogram
- **return_samples** (*bool, optional*) – see return-value of documentation
- **n_cpu** (*int, optional*) – number of CPUs to use for multiprocessing, defaults to all available CPUs

### Example

draw samples from multivariate normal distribution: draw_samples = lambda size: np.random.multivariate_normal(x, Ux, size)

**build a function, that only accepts one argument by masking addtional kwargs:** evaluate = functools.partial(_UMCevaluate, nbb=b.size, x=x, Delta=Delta, phi=phi, theta=theta, sigma=sigma, blow=blow, alow=alow) evaluate = functools.partial(bigFunction, **dict_of_kwargs)

If `return_samples` is `False`, the method returns:

**Returns**

- **y** (*np.ndarray*) – filter output signal
- **Uy** (*np.ndarray*) – uncertainty associated with
- **happr** (*dict*) – dictionary of bin-edges and bin-counts

Otherwise the method returns:

**Returns** **sims** – dict of samples and corresponding results of every evaluated simulation

**Return type** dict

### References


- Eichstädt, Link, Harris, Elster [Eichst2012]

# CHAPTER 3

# Design of deconvolution filters

The estimation of the measurand in the analysis of dynamic measurements typically corresponds to a deconvolution problem. Therefore, a digital filter can be designed whose input is the measured system output signal and whose output is an estimate of the measurand. This module implements methods for the design of such filters given an array of frequency response values with associated uncertainties for the measurement system.

This module contains functions to carry out a least-squares fit of a digital filter to the reciprocal of a given complex frequency response.

This module contains the following functions:

- *LSFIR*: Least-squares fit of a digital FIR filter to the reciprocal of a given frequency response.
- *LSFIR_unc*: Design of FIR filter as fit to reciprocal of frequency response values with uncertainty
- *LSFIR_uncMC*: Design of FIR filter as fit to reciprocal of frequency response values with uncertainty via Monte Carlo
- *LSIIR*: Design of a stable IIR filter as fit to reciprocal of frequency response values
- *LSIIR_unc*: Design of a stable IIR filter as fit to reciprocal of frequency response values with uncertainty

Deprecated since version 1.2.71: The module *deconvolution* will be combined with the module *identification* and renamed to *model_estimation* in the next major release 3.0. From then on you should only use the new module *model_estimation* instead.

PyDynamic.deconvolution.fit_filter.**LSFIR**(*H*, *N*, *tau*, *f*, *Fs*, *Wt=None*)

Least-squares fit of a digital FIR filter to the reciprocal of a given frequency response.

**Parameters**

- **H** (*np.ndarray of shape (M,) and dtype complex*) – frequency response values
- **N** (*int*) – FIR filter order
- **tau** (*float*) – delay of filter
- **f** (*np.ndarray of shape (M,)*) – frequencies
- **Fs** (*float*) – sampling frequency of digital filter

- **Wt** (*np.ndarray of shape (M,) – optional*) – vector of weights

**Returns** **bFIR** – filter coefficients

**Return type** np.ndarray of shape (N,)

### References

- Elster and Link [Elster2008]

PyDynamic.deconvolution.fit_filter.**LSFIR_unc**(*H*, *UH*, *N*, *tau*, *f*, *Fs*, *wt=None*, *verbose=True*, *trunc_svd_tol=None*)

Design of FIR filter as fit to reciprocal of frequency response values with uncertainty

Least-squares fit of a digital FIR filter to the reciprocal of a frequency response for which associated uncertainties are given for its real and imaginary part. Uncertainties are propagated using a truncated svd and linear matrix propagation.

**Parameters**

- **H** (*np.ndarray of shape (M,)*) – frequency response values
- **UH** (*np.ndarray of shape (2M,2M)*) – uncertainties associated with the real and imaginary part
- **N** (*int*) – FIR filter order
- **tau** (*float*) – delay of filter
- **f** (*np.ndarray of shape (M,)*) – frequencies
- **Fs** (*float*) – sampling frequency of digital filter
- **wt** (*np.ndarray of shape (2M,) – optional*) – array of weights for a weighted least-squares method
- **verbose** (*bool, optional*) – whether to print statements to the command line
- **trunc_svd_tol** (*float*) – lower bound for singular values to be considered for pseudo-inverse

**Returns**

- **b** (*np.ndarray of shape (N+1,)*) – filter coefficients of shape
- **Ub** (*np.ndarray of shape (N+1,N+1)*) – uncertainties associated with b

### References

- Elster and Link [Elster2008]

PyDynamic.deconvolution.fit_filter.**LSIIR**(*Hvals*, *Nb*, *Na*, *f*, *Fs*, *tau*, *justFit=False*, *verbose=True*)

Design of a stable IIR filter as fit to reciprocal of frequency response values

Least-squares fit of a digital IIR filter to the reciprocal of a given set of frequency response values using the equation-error method and stabilization by pole mapping and introduction of a time delay.

**Parameters**

- **Hvals** (*np.ndarray of shape (M,) and dtype complex*) – frequency response values.

- **Nb** (*int*) – order of IIR numerator polynomial.
- **Na** (*int*) – order of IIR denominator polynomial.
- **f** (*np.ndarray of shape (M,)*) – frequencies corresponding to Hvals
- **Fs** (*float*) – sampling frequency for digital IIR filter.
- **tau** (*float*) – initial estimate of time delay for filter stabilization.
- **justFit** (*bool*) – if True then no stabilization is carried out.
- **verbose** (*bool*) – If True print some more detail about input parameters.

**Returns**

- **b, a** (*np.ndarray*) – IIR filter coefficients
- **tau** (*int*) – time delay (in samples)

### References

- Eichstädt, Elster, Esward, Hessling [Eichst2010]

PyDynamic.deconvolution.fit_filter.**LSIIR_unc**(*H*, *UH*, *Nb*, *Na*, *f*, *Fs*, *tau=0*)

Design of stabel IIR filter as fit to reciprocal of given frequency response with uncertainty

Least-squares fit of a digital IIR filter to the reciprocal of a given set of frequency response values with given associated uncertainty. Propagation of uncertainties is carried out using the Monte Carlo method.

**Parameters**

- **H** (*np.ndarray of shape (M,) and dtype complex*) – frequency response values.
- **UH** (*np.ndarray of shape (2M,2M)*) – uncertainties associated with real and imaginary part of H
- **Nb** (*int*) – order of IIR numerator polynomial.
- **Na** (*int*) – order of IIR denominator polynomial.
- **f** (*np.ndarray of shape (M,)*) – frequencies corresponding to H
- **Fs** (*float*) – sampling frequency for digital IIR filter.
- **tau** (*float*) – initial estimate of time delay for filter stabilization.

**Returns**

- **b,a** (*np.ndarray*) – IIR filter coefficients
- **tau** (*int*) – time delay (in samples)
- **Uba** (*np.ndarray of shape (Nb+Na+1, Nb+Na+1)*) – uncertainties associated with [a[1:],b]

### References

- Eichstädt, Elster, Esward and Hessling [Eichst2010]

**See also:**

*PyDynamic.uncertainty.propagate_filter.IIRuncFilter* *PyDynamic.deconvolution.fit_filter.LSIIR*

PyDynamic.deconvolution.fit_filter.**LSFIR_uncMC**(*H*, *UH*, *N*, *tau*, *f*, *Fs*, *verbose=True*)

Design of FIR filter as fit to reciprocal of frequency response values with uncertainty

Least-squares fit of a FIR filter to the reciprocal of a frequency response for which associated uncertainties are given for its real and imaginary parts. Uncertainties are propagated using a Monte Carlo method. This method may help in cases where the weighting matrix or the Jacobian are ill-conditioned, resulting in false uncertainties associated with the filter coefficients.

**Parameters**

- **H** (*np.ndarray of shape (M,) and dtype complex*) – frequency response values
- **UH** (*np.ndarray of shape (2M,2M)*) – uncertainties associated with the real and imaginary part of H
- **N** (*int*) – FIR filter order
- **tau** (*int*) – time delay of filter in samples
- **f** (*np.ndarray of shape (M,)*) – frequencies corresponding to H
- **Fs** (*float*) – sampling frequency of digital filter
- **verbose** (*bool, optional*) – whether to print statements to the command line

**Returns**

- **b** (*np.ndarray of shape (N+1,)*) – filter coefficients of shape
- **Ub** (*np.ndarray of shape (N+1, N+1)*) – uncertainties associated with b

### References


- Elster and Link [Elster2008]

# CHAPTER 4

# Miscellaneous

## 4.1 Tools for 2nd order systems

A collection of modules and methods that are used throughout the whole package. Methods specialized for second order dynamic systems, such as the ones used for high-class accelerometers.

This module contains the following functions:

- *sos_FreqResp*: Calculation of the system frequency response
- *sos_phys2filter*: Calculation of continuous filter coefficients from physical parameters
- *sos_absphase*: Propagation of uncertainty from physical parameters to real and imaginary part of system's transfer function using GUM S2 Monte Carlo
- *sos_realimag*: Propagation of uncertainty from physical parameters to real and imaginary part of system's transfer function using GUM S2 Monte Carlo

PyDynamic.misc.SecondOrderSystem.**sos_FreqResp**(*S*, *d*, *f0*, *freqs*)

Calculation of the system frequency response

The frequency response is calculated from the continuous physical model of a second order system given by

$H(f) = \frac{4S\pi^2 f_0^2}{(2\pi f_0)^2 + 2jd(2\pi f_0)f - f^2}$

If the provided system parameters are vectors then $H(f)$ is calculated for each set of parameters. This is helpful for Monte Carlo simulations by using draws from the model parameters

**Parameters**

- **S** (*float or ndarray shape (K,)*) – static gain
- **d** (*float or ndarray shape (K,)*) – damping parameter
- **f0** (*float or ndarray shape (K,)*) – resonance frequency
- **freqs** (*ndarray shape (N,)*) – frequencies at which to calculate the freq response

**Returns** **H** – complex frequency response values

**Return type** ndarray shape (N,) or ndarray shape (N,K)

PyDynamic.misc.SecondOrderSystem.**sos_phys2filter**(*S*, *d*, *f0*)

Calculation of continuous filter coefficients from physical parameters.

If the provided system parameters are vectors then the filter coefficients are calculated for each set of parameters. This is helpful for Monte Carlo simulations by using draws from the model parameters

**Parameters**

- **S** (*float*) – static gain
- **d** (*float*) – damping parameter
- **f0** (*float*) – resonance frequency

**Returns** **b,a** – analogue filter coefficients

**Return type** ndarray

PyDynamic.misc.SecondOrderSystem.**sos_absphase**(*S*, *d*, *f0*, *uS*, *ud*, *uf0*, *f*, *runs=10000*)

Propagation of uncertainty from physical parameters to real and imaginary part of system's transfer function using GUM S2 Monte Carlo.

**Parameters**

- **S** (*float*) – static gain
- **d** (*float*) – damping
- **f0** (*float*) – resonance frequency
- **uS** (*float*) – uncertainty associated with static gain
- **ud** (*float*) – uncertainty associated with damping
- **uf0** (*float*) – uncertainty associated with resonance frequency
- **f** (*ndarray, shape (N,)*) – frequency values at which to calculate amplitue and phase

**Returns**

- **Hmean** (*ndarray, shape (N,)*) – best estimate of complex frequency response values
- **Hcov** (*ndarray, shape (2N,2N)*) – covariance matrix [ [u(abs,abs), u(abs,phase)], [u(phase,abs), u(phase,phase)] ]

PyDynamic.misc.SecondOrderSystem.**sos_realimag**(*S*, *d*, *f0*, *uS*, *ud*, *uf0*, *f*, *runs=10000*)

Propagation of uncertainty from physical parameters to real and imaginary part of system's transfer function using GUM S2 Monte Carlo.

**Parameters**

- **S** (*float*) – static gain
- **d** (*float*) – damping
- **f0** (*float*) – resonance frequency
- **uS** (*float*) – uncertainty associated with static gain
- **ud** (*float*) – uncertainty associated with damping
- **uf0** (*float*) – uncertainty associated with resonance frequency
- **f** (*ndarray, shape (N,)*) – frequency values at which to calculate real and imaginary part

**Returns**

- **Hmean** (*ndarray, shape (N,)*) – best estimate of complex frequency response values
- **Hcov** (*ndarray, shape (2N,2N)*) – covariance matrix [ [u(real,real), u(real,imag)], [u(imag,real), u(imag,imag)] ]

## 4.2 Tools for digital filters

A collection of methods which are related to filter design.

This module contains the following functions:

- *db*: Calculation of decibel values $20\log_{10}(x)$ for a vector of values
- *ua*: Shortcut for calculation of unwrapped angle of complex values
- *grpdelay*: Calculation of the group delay of a digital filter
- *mapinside*: Maps the roots of polynomial with coefficients $a$ to the unit circle
- *kaiser_lowpass*: Design of a FIR lowpass filter using the window technique with a Kaiser window.
- *isstable*: Determine whether a given IIR filter is stable
- *savitzky_golay*: Smooth (and optionally differentiate) data with a Savitzky-Golay filter

PyDynamic.misc.filterstuff.**grpdelay**(*b*, *a*, *Fs*, *nfft=512*)

Calculation of the group delay of a digital filter

**Parameters**

- **b** (*ndarray*) – IIR filter numerator coefficients
- **a** (*ndarray*) – IIR filter denominator coefficients
- **Fs** (*float*) – sampling frequency of the filter
- **nfft** (*int*) – number of FFT bins

**Returns**

- **group_delay** (*np.ndarray*) – group delay values
- **frequencies** (*ndarray*) – frequencies at which the group delay is calculated

### References

- Smith, online book [Smith]

PyDynamic.misc.filterstuff.**mapinside**(*a*)

Maps the roots of polynomial to the unit circle.

Maps the roots of polynomial with coefficients $a$ to the unit circle.

**Parameters** **a** (*ndarray*) – polynomial coefficients

**Returns** **a** – polynomial coefficients with all roots inside or on the unit circle

**Return type** ndarray

PyDynamic.misc.filterstuff.**kaiser_lowpass**(*L*, *fcut*, *Fs*, *beta=8.0*)
Design of a FIR lowpass filter using the window technique with a Kaiser window.

This method uses a Kaiser window. Filters of that type are often used as FIR low-pass filters due to their linear phase.

**Parameters**

- **L** (*int*) – filter order (window length)
- **fcut** (*float*) – desired cut-off frequency
- **Fs** (*float*) – sampling frequency
- **beta** (*float*) – scaling parameter for the Kaiser window

**Returns**

- **blow** (*ndarray*) – FIR filter coefficients
- **shift** (*int*) – delay of the filter (in samples)

PyDynamic.misc.filterstuff.**isstable**(*b*, *a*, *ftype='digital'*)
Determine whether *IIR filter (b,a)* is stable

Determine whether *IIR filter (b,a)* is stable by checking roots of the polynomial ´a´.

**Parameters**

- **b** (*ndarray*) – filter numerator coefficients
- **a** (*ndarray*) – filter denominator coefficients
- **ftype** (*string*) – type of filter (*digital* or *analog*)

**Returns** **stable** – whether filter is stable or not

**Return type** bool

PyDynamic.misc.filterstuff.**savitzky_golay**(*y*, *window_size*, *order*, *deriv=0*, *delta=1.0*)
Smooth (and optionally differentiate) data with a Savitzky-Golay filter

The Savitzky-Golay filter removes high frequency noise from data. It has the advantage of preserving the original shape and features of the signal better than other types of filtering approaches, such as moving averages techniques.

Source obtained from scipy cookbook (online), downloaded 2013-09-13

**Parameters**

- **y** (*ndarray, shape (N,)*) – the values of the time history of the signal
- **window_size** (*int*) – the length of the window. Must be an odd integer number
- **order** (*int*) – the order of the polynomial used in the filtering. Must be less then *window_size* - 1.
- **deriv** (*int, optional*) – The order of the derivative to compute. This must be a nonnegative integer. The default is 0, which means to filter the data without differentiating.
- **delta** (*float, optional*) – The spacing of the samples to which the filter will be applied. This is only used if deriv > 0. This includes a factor $n!/h^n$, where $n$ is represented by *deriv* and $1/h$ by *delta*.

**Returns** **ys** – the smoothed signal (or it's n-th derivative).

**Return type** ndarray, shape (N,)

### Notes

The Savitzky-Golay is a type of low-pass filter, particularly suited for smoothing noisy data. The main idea behind this approach is to make for each point a least-square fit with a polynomial of high order over a odd-sized window centered at the point.

### References


- Savitzky et al. [Savitzky]
- Numerical Recipes [NumRec]


## 4.3 Test signals

Collection of test signals which can be used to simulate dynamic measurements and test methods.

This module contains the following functions:

- *shocklikeGaussian*: signal that resembles a shock excitation as a Gaussian followed by a smaller Gaussian of opposite sign
- *GaussianPulse*: Generates a Gaussian pulse at $t_0$ with height $m_0$ and std $sigma$
- *rect*: Rectangular signal of given height and width $t_1 - t_0$
- *squarepulse*: Generates a series of rect functions to represent a square pulse signal
- *sine*: Generate a sine signal

PyDynamic.misc.testsignals.**shocklikeGaussian**(*time*, *t0*, *m0*, *sigma*, *noise=0.0*)
Generates a signal that resembles a shock excitation as a Gaussian followed by a smaller Gaussian of opposite sign.

**Parameters**

- **time** (*np.ndarray of shape (N,)*) – time instants (equidistant)
- **t0** (*float*) – time instant of signal maximum
- **m0** (*float*) – signal maximum
- **sigma** (*float*) – std of main pulse
- **noise** (*float, optional*) – std of simulated signal noise

**Returns** **x** – signal amplitudes at time instants

**Return type** np.ndarray of shape (N,)

PyDynamic.misc.testsignals.**GaussianPulse**(*time*, *t0*, *m0*, *sigma*, *noise=0.0*)
Generates a Gaussian pulse at t0 with height m0 and std sigma

**Parameters**

- **time** (*np.ndarray of shape (N,)*) – time instants (equidistant)
- **t0** (*float*) – time instant of signal maximum
- **m0** (*float*) – signal maximum
- **sigma** (*float*) – std of pulse
- **noise** (*float, optional*) – std of simulated signal noise

**Returns** **x** – signal amplitudes at time instants

**Return type** np.ndarray of shape (N,)

PyDynamic.misc.testsignals.**rect**(*time*, *t0*, *t1*, *height=1*, *noise=0.0*)
Rectangular signal of given height and width t1-t0

**Parameters**

- **time** (*np.ndarray of shape (N,)*) – time instants (equidistant)
- **t0** (*float*) – time instant of rect lhs
- **t1** (*float*) – time instant of rect rhs
- **height** (*float*) – signal maximum
- **noise** (*float, optional*) – std of simulated signal noise

**Returns** **x** – signal amplitudes at time instants

**Return type** np.ndarray of shape (N,)

PyDynamic.misc.testsignals.**squarepulse**(*time*, *height*, *numpulse=4*, *noise=0.0*)
Generates a series of rect functions to represent a square pulse signal

**Parameters**

- **time** (*np.ndarray of shape (N,)*) – time instants
- **height** (*float*) – height of the rectangular pulses
- **numpulse** (*int*) – number of pulses
- **noise** (*float, optional*) – std of simulated signal noise

**Returns** **x** – signal amplitude at time instants

**Return type** np.ndarray of shape (N,)

**class** PyDynamic.misc.testsignals.**corr_noise**(*w*, *sigma*, *seed=None*)
Base class for generation of a correlated noise process.

PyDynamic.misc.testsignals.**sine**(*time*, *amp=1.0*, *freq=6.283185307179586*, *noise=0.0*)
Generate a sine signal

**Parameters**

- **time** (*np.ndarray of shape (N,)*) – time instants
- **amp** (*float, optional*) – amplitude of the sine (default = 1.0)
- **freq** (*float, optional*) – frequency of the sine in Hz (default = $2 * \pi$)
- **noise** (*float, optional*) – std of simulated signal noise (default = 0.0)

**Returns** **x** – signal amplitude at time instants

**Return type** np.ndarray of shape (N,)

## 4.4 Noise related functions

PyDynamic.misc.noise.**ARMA**(*length*, *phi=0.0*, *theta=0.0*, *std=1.0*)
Generate time-series of a predefined ARMA-process based on this equation: $\sum_{j=1}^{\min(p,n-1)} \phi_j \epsilon[n-j] +$

$\sum_{j=1}^{\min(q,n-1)} \theta_j w[n-j]$ where w is white gaussian noise. Equation and algorithm taken from [Eichst2012] .

**Parameters**

- **length** (*int*) – how long the drawn sample will be
- **phi** (*float, list or numpy.ndarray, shape (p, )*) – AR-coefficients
- **theta** (*float, list or numpy.ndarray*) – MA-coefficients
- **std** (*float*) – std of the gaussian white noise that is feeded into the ARMA-model

**Returns** **e** – time-series of the predefined ARMA-process

**Return type** numpy.ndarray, shape (length, )

### References


- Eichstädt, Link, Harris and Elster [Eichst2012]


## 4.5 Miscellaneous useful helper functions

Collection of miscellaneous helper functions.

This module contains the following functions:

- *print_vec*: Print vector (1D array) to the console or return as formatted string
- *print_mat*: Print matrix (2D array) to the console or return as formatted string
- *make_semiposdef*: Make quadratic matrix positive semi-definite
- *FreqResp2RealImag*: Calculate real and imaginary parts from frequency response
- *make_equidistant*: Interpolate non-equidistant time series to equidistant

PyDynamic.misc.tools.**print_mat**(*matrix*, *prec=5*, *vertical=False*, *retS=False*)
Print matrix (2D array) to the console or return as formatted string

**Parameters**

- **matrix** (*(M,N) array_like*) –
- **prec** (*int*) – the precision of the output
- **vertical** (*bool*) – print out vertical or not
- **retS** (*bool*) – print or return string

**Returns** **s** – if retS is True

**Return type** str

PyDynamic.misc.tools.**print_vec**(*vector*, *prec=5*, *retS=False*, *vertical=False*)
Print vector (1D array) to the console or return as formatted string

**Parameters**

- **vector** (*(M,) array_like*) –
- **prec** (*int*) – the precision of the output
- **vertical** (*bool*) – print out vertical or not

- **retS** (*bool*) – print or return string

**Returns** **s** – if retS is True

**Return type** str

PyDynamic.misc.tools.**make_semiposdef**(*matrix*, *maxiter=10*, *tol=1e-12*, *verbose=False*)
Make quadratic matrix positive semi-definite by increasing its eigenvalues

**Parameters**

- **matrix** (*(N,N) array_like*) –
- **maxiter** (*int*) – the maximum number of iterations for increasing the eigenvalues
- **tol** (*float*) – tolerance for deciding if pos. semi-def.
- **verbose** (*bool*) – If True print some more detail about input parameters.

**Returns** quadratic positive semi-definite matrix

**Return type** (N,N) array_like

PyDynamic.misc.tools.**FreqResp2RealImag**(*Abs*, *Phase*, *Unc*, *MCruns=10000.0*)
Calculate real and imaginary parts from frequency response

Calculate real and imaginary parts from amplitude and phase with associated uncertainties.

**Parameters**

- **Abs** (*(N,) array_like*) – amplitude values
- **Phase** (*(N,) array_like*) – phase values in rad
- **Unc** (*(2N, 2N) or (2N,) array_like*) – uncertainties
- **MCruns** (*bool*) – Iterations for Monte Carlo simulation

**Returns**

- **Re, Im** (*(N,) array_like*) – real and imaginary parts (best estimate)
- **URI** (*(2N, 2N) array_like*) – uncertainties assoc. with Re and Im

PyDynamic.misc.tools.**make_equidistant**(*t*, *y*, *uy*, *dt=0.05*, *kind='linear'*)
Interpolate non-equidistant time series to equidistant

Interpolate measurement values and propagate uncertainties accordingly.

**Parameters**

- **t** (*(N,) array_like*) – timestamps in ascending order
- **y** (*(N,) array_like*) – corresponding measurement values
- **uy** (*(N,) array_like*) – corresponding measurement values' uncertainties
- **dt** (*float, optional*) – desired interval length in seconds
- **kind** (*str, optional*) – Specifies the kind of interpolation for the measurement values as a string ('previous', 'next', 'nearest' or 'linear').

**Returns**

- **t_new** (*(N,) array_like*) – timestamps
- **y_new** (*(N,) array_like*) – measurement values
- **uy_new** (*(N,) array_like*) – measurement values' uncertainties

### References

- White [White2017]

PyDynamic.misc.tools.**progress_bar**(*count*, *count_max*, *width=30*, *prefix=''*, *done_indicator='#'*, *todo_indicator='.'*, *fout=<_io.TextIOWrapper name='<stdout>' mode='w' encoding='UTF-8'>*)

A simple and reusable progress-bar.

**Parameters**

- **count** (*int*) – current status of iterations, assumed to be zero-based
- **count_max** (*int*) – total number of iterations
- **width** (*int, optional*) – width of the actual progressbar (actual printed line will be wider)
- **prefix** (*str, optional*) – some text that will be printed in front of the bar (i.e. "Progress of ABC:")
- **done_indicator** (*str, optional*) – what character is used as "already-done"-indicator
- **todo_indicator** (*str, optional*) – what character is used as "not-done-yet"-indicator
- **fout** (*file-object, optional*) – where the progress-bar should be written/printed to

CHAPTER 5

# Indices and tables

- genindex
- modindex
- search

CHAPTER 6

# References

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# Python Module Index

## p

PyDynamic.deconvolution.fit_filter, 39
PyDynamic.misc.filterstuff, 45
PyDynamic.misc.noise, 48
PyDynamic.misc.SecondOrderSystem, 43
PyDynamic.misc.testsignals, 47
PyDynamic.misc.tools, 49
PyDynamic.uncertainty.propagate_DFT, 27
PyDynamic.uncertainty.propagate_filter, 32
PyDynamic.uncertainty.propagate_MonteCarlo, 33

# Index


## A

AmpPhase2DFT() (*in module PyDynamic.uncertainty.propagate_DFT*), 30
AmpPhase2Time() (*in module PyDynamic.uncertainty.propagate_DFT*), 31
ARMA() (*in module PyDynamic.misc.noise*), 48

## C

corr_noise (*class in PyDynamic.misc.testsignals*), 48

## D

DFT2AmpPhase() (*in module PyDynamic.uncertainty.propagate_DFT*), 30
DFT_deconv() (*in module PyDynamic.uncertainty.propagate_DFT*), 29
DFT_multiply() (*in module PyDynamic.uncertainty.propagate_DFT*), 30
DFT_transferfunction() (*in module PyDynamic.uncertainty.propagate_DFT*), 29

## F

FIRuncFilter() (*in module PyDynamic.uncertainty.propagate_filter*), 32
FreqResp2RealImag() (*in module PyDynamic.misc.tools*), 50

## G

GaussianPulse() (*in module PyDynamic.misc.testsignals*), 47
grpdelay() (*in module PyDynamic.misc.filterstuff*), 45
GUM_DFT() (*in module PyDynamic.uncertainty.propagate_DFT*), 28
GUM_DFTfreq() (*in module PyDynamic.uncertainty.propagate_DFT*), 29
GUM_iDFT() (*in module PyDynamic.uncertainty.propagate_DFT*), 28

## I

IIRuncFilter() (*in module PyDynamic.uncertainty.propagate_filter*), 33
isstable() (*in module PyDynamic.misc.filterstuff*), 46

## K

kaiser_lowpass() (*in module PyDynamic.misc.filterstuff*), 45

## L

LSFIR() (*in module PyDynamic.deconvolution.fit_filter*), 39
LSFIR_unc() (*in module PyDynamic.deconvolution.fit_filter*), 40
LSFIR_uncMC() (*in module PyDynamic.deconvolution.fit_filter*), 41
LSIIR() (*in module PyDynamic.deconvolution.fit_filter*), 40
LSIIR_unc() (*in module PyDynamic.deconvolution.fit_filter*), 41

## M

make_equidistant() (*in module PyDynamic.misc.tools*), 50
make_semiposdef() (*in module PyDynamic.misc.tools*), 50
mapinside() (*in module PyDynamic.misc.filterstuff*), 45
MC() (*in module PyDynamic.uncertainty.propagate_MonteCarlo*), 33

## P

print_mat() (*in module PyDynamic.misc.tools*), 49
print_vec() (*in module PyDynamic.misc.tools*), 49
progress_bar() (*in module PyDynamic.misc.tools*), 51

PyDynamic.deconvolution.fit_filter (*module*), 39
PyDynamic.misc.filterstuff (*module*), 45
PyDynamic.misc.noise (*module*), 48
PyDynamic.misc.SecondOrderSystem (*module*), 43
PyDynamic.misc.testsignals (*module*), 47
PyDynamic.misc.tools (*module*), 49
PyDynamic.uncertainty.propagate_DFT (*module*), 27
PyDynamic.uncertainty.propagate_filter (*module*), 32
PyDynamic.uncertainty.propagate_MonteCarlo (*module*), 33

## R

rect() (*in module PyDynamic.misc.testsignals*), 48

## S

savitzky_golay() (*in module PyDynamic.misc.filterstuff*), 46
shocklikeGaussian() (*in module PyDynamic.misc.testsignals*), 47
sine() (*in module PyDynamic.misc.testsignals*), 48
SMC() (*in module PyDynamic.uncertainty.propagate_MonteCarlo*), 34
sos_absphase() (*in module PyDynamic.misc.SecondOrderSystem*), 44
sos_FreqResp() (*in module PyDynamic.misc.SecondOrderSystem*), 43
sos_phys2filter() (*in module PyDynamic.misc.SecondOrderSystem*), 44
sos_realimag() (*in module PyDynamic.misc.SecondOrderSystem*), 44
squarepulse() (*in module PyDynamic.misc.testsignals*), 48

## T

Time2AmpPhase() (*in module PyDynamic.uncertainty.propagate_DFT*), 31
Time2AmpPhase_multi() (*in module PyDynamic.uncertainty.propagate_DFT*), 31

## U

UMC() (*in module PyDynamic.uncertainty.propagate_MonteCarlo*), 35
UMC_generic() (*in module PyDynamic.uncertainty.propagate_MonteCarlo*), 36